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Paramagnetism of Aharonov-Bohm and Direct Responses

*Sang-Shik Oh**

Aharonov-Bohm 반응과 직접반응에서의 상자성

오 상 식*

Summary

Following recent works on the Aharonov-Bohm (AB) response of canonical ensembles, we obtain the exact expression of the persistent current in a mesoscopic metallic ring and evaluate the disorder-averaged orbital paramagnetic susceptibility of an isolated metallic disk. As in the AB case, we find that $\chi_{para} \sim |\chi_L| (E_c/\max\{2\pi T, \gamma\})$ in two dimensions. This is larger than the disorder-averaged diamagnetic susceptibility of a grand canonical ensemble, $\chi_{dia} \sim \chi_L$, up to the characteristic flux, $\phi_c \sim \phi_0 (E_c/\max\{2\pi T, \gamma\})^{-1/2}$. In stronger fields $\phi_c \leq \phi \leq \phi_0/2$, the magnetic susceptibility of the direct response reduces to the value of order χ_L

Introduction

The transport properties of submicron-size metallic samples at low temperature have been shown to exhibit features of the quantum coherence of the electron wave function along the whole sample. For such "mesoscopic" systems (Spivak and Zyuzin, 1991), some magnetic field-related effects, like Aharonov-Bohm (AB) oscillation in the resistance (Stone and Imry, 1986; Aronov and Sharvin, 1987) and the persistent current of a narrow ring (Büttiker *et al.*, 1983) pierced by the magnetic field, have been received considerable consideration. After the first experimental evidence (Levy *et al.*, 1990; Chandrasekar *et al.*, 1991) for existance of the persistent current was reported, there have been a lot of theoretical attempts (Ambegaokar and Eckern, 1990; Schumid, 1991; Altshuler *et al.*, 1991; Efetov, 1991; Kopietz, 1992) to analyze the experiment of Levy *et al.* Schmid, Altshuler, Gefen and Imry have shown that in a canonical ensemble the persistent Aharonov-Bohm (AB)

* 자연과학대학 물리학과(Dept. of Physics, Cheju Univ., Cheju-do, 690-756, Korea)

currents have, on the average, the paramagnetic sign at zero temperature. This phenomenon has been attributed to correlations of energy levels in a disordered metal (Altshuler and Shklovskii, 1986) and the underlying physical picture can be summarized as follows. Whereas a standard derivation of the response function assumes fixed chemical potential, it is only appropriate for the circumstance of a conductor connected to the electron reservoirs with the well-defined chemical potentials. However, this is not the typical case for the magnetic response, in which a sample is usually isolated from the electron reservoir. The difference between the grand canonical and canonical ensembles is that in the latter the total number of electrons in the system remains constant, while in the former the electron closest to the Fermi level can leave or enter the sample once its energy crosses the chemical potential with the change of the magnetic flux through the hole of the ring (AB flux). Therefore, AB current is a single level current and has the same (paramagnetic) sign for the levels crossing the chemical potential from above and below since those levels have slopes of the opposite signs as a function of the AB flux (Trivedi and Browne, 1988). The aim of this paper is to present the mesoscopic aspects of magnetic response of a small metallic systems with normal impurities; we do not consider the effects of magnetic impurities, such as spin-flip and spin-orbital scattering. We derive an exact expression of the AB current of the mesoscopic metallic ring with finite temperature and expand the methods of Schmid (1991) and Altshuler *et al.*, (1991) to the direct magnetic response of a canonical ensemble, that is the response to the flux through the metal, and compare it to the AB response. The size of the sample L is assumed to be the circumference of the two dimensional ring for the AB response and of the two dimensional disk for the direct response, respectively. For a ring or a disk of thickness $a \ll L$, the expressions for the magnetic moment and susceptibility have an extra factor of $(k_F a)^{-2}$, where k_F is the fermi wave number. We also assume that the inelastic level broadening γ, and energy associated with temperature are larger than the average interlevel spacing of a sample $\Delta = [N(0)V]^{-1}$, being $N(0)$ the single-particle density of states at the Fermi level and V the volume (area) of the sample. This paper is organized in the following sequence. In Chapter, Ⅱ, we briefly outline the derivation of Schmid and Altshuler *et al.*, The novel of our analysis is the exact expression for the persistent current. In Chapter Ⅲ, we derive the expression for the average paramagnetic susceptibility and the correlation function of the chemical potential of an isolated metallic disk and establish the complete analogy between the AB (flux-periodic) and the zero mode forms of the Cooper propagator (Cooperon). In conclusion, we outline the magnetic moment and susceptibility for the AB and direct responses and discuss the similarity and difference of both cases.

Aharonov-Bohm Response

We begin with the derivation of the paramagnetic component of the AB (persistent) current in a narrow ring. It was shown that the average free energy F of the canonical ensemble can be expressed in terms of the average free energy Ω of the identically prepared grand canonical ensemble as

$$F = \Omega - \frac{1}{2}\frac{\partial^2 \Omega}{\partial \langle\mu\rangle^2}\langle(\delta\mu)^2\rangle = \Omega - \frac{1}{2}\Delta^{-1}\langle(\delta\mu)^2\rangle. \tag{1}$$

Here $\langle\mu\rangle$ is the average chemical potential of the canonical ensemble, which is also the true chemical potential of the corresponding grand canonical ensemble, $\langle(\delta\mu)^2\rangle=\langle(\mu-\langle\mu\rangle)^2\rangle$ is its average variance. The latter can be rewritten as

$$\langle(\delta\mu)^2\rangle=[N(0)]^{-2} \iint \frac{d\epsilon_1}{2\pi}\frac{d\epsilon_2}{2\pi}\langle N(\epsilon_1)N(\epsilon_2)\rangle f(\epsilon_1)f(\epsilon_2). \quad (2)$$

where $f(\epsilon)$ is the fermi distribution function and the correlation function of the density of states is given by (Altshuler and Shklovskii, 1986)

$$\langle N(\epsilon_1)N(\epsilon_2)\rangle = \frac{s^2}{2\pi^2V^2}\sum_q \mathrm{Re}[-i(\epsilon_1-\epsilon_2)+\gamma+Dq^2]^{-2}. \quad (3)$$

In Eq. (3) s, D are the spin degrees of freedom and the diffusion coefficient, respectively and q is the momentum of the Cooperon. In a thin ring the transverse momenta can be omitted while the longitudinal momentum can be expressed in terms of the circumference of the ring as

$$q_1=(\pi/L)(n+2\phi_{AB}/\phi_0), \quad (4)$$

where n is any integer and ϕ_0 is the flux quantum. Such dependence on the flux is inherent to the Cooperon: the diffusion propergator, on the other hand, does not depend on the flux and is omitted here. Combining Eqs. (1)-(4), we obtain

$$F-\Omega=\frac{s^2\Delta}{(4\pi)^2}\iint d\epsilon_1)f(\epsilon_2)\sum_{n=-\infty}^{\infty} \mathrm{Re}[(\epsilon_1-\epsilon_2)+i\gamma+i\pi^2E_c(n+\frac{2\phi_{AB}}{\phi_0}]^{-2}. \quad (5)$$

where $E_c=\hbar D/L^2$, being much larger than max$\{2\pi T,\gamma\}$, is the maximal energy for the occurance of the quantum coherence. Two observations should be made about Eq. (5). First, the right-hand side (r.h.s.) diverges at zero flux. However, we are concerned only with the magnetic part of the free energy and hence will neglect the divergent part as an irrelevant constant. Second, the r.h.s. is clearly periodic with the period of the half the flux quantum. One can calculate the AB current from the thermodynamics (Byers and Yang, 1961), putting $c=1$

$$I(\phi)=-\frac{\partial F}{\partial\phi}. \quad (6)$$

Converting the integral into a Matsubara sum and then evaluating the sum over n first with the identity:

$$\sum_{n=-\infty}^{\infty}\frac{1}{(n+a)^2+b^2}=\frac{\pi}{b}\frac{\sinh 2\pi b}{\cosh 2\pi b-\cos 2\pi a}. \quad (7)$$

the following expression for the persistent current is given

$$I=\frac{2s^2}{(2\pi)^4}\frac{e\Delta}{\hbar}\sin\left(\frac{4\pi\phi_{AB}}{\phi_0}\right)\left(\frac{2\pi T}{2\pi T+\gamma}\frac{\alpha\sinh\alpha}{(\cosh\alpha-\cos(4\pi\phi_{AB}/\phi_0))^2}+\frac{1}{\cosh\alpha-\cos(4\pi\phi_{AB}/\phi_0)}\right). \quad (8)$$

where $\alpha=\sqrt{(2\pi T+\gamma)/4E_c}$. In the linear response approximation, $2\phi_{AB}\le\phi_c$, the limits of Eq. (8) are as follows:

$$I\sim\begin{cases}\frac{3s^2}{2\pi^3}\frac{e\Delta}{\hbar}\frac{E_c}{2\pi T}\frac{2\phi_{AB}}{\phi_0}=\frac{3s^2}{2\pi^3}\frac{e\Delta}{h}\sqrt{\frac{E_c}{2\pi T}}\frac{2\phi_{AB}}{\phi_c}, & E_c\gg 2\pi T\gg\gamma \\ \frac{s^2}{2\pi^3}\frac{e\Delta}{h}\frac{E_c}{\gamma}\frac{2\phi_{AB}}{\phi_0}=\frac{s^2}{2\pi^3}\frac{e\Delta}{h}\sqrt{\frac{E_c}{\gamma}}\frac{2\phi_{AB}}{\phi_c}, & E_c\gg\gamma\gg 2\pi T\end{cases} \quad (8')$$

When max$\{2\pi T,\gamma\}\le\Delta$, the quantity of order Δ should be substitnted for $2\pi T$ and γ in the above equations (Altshuler *et al.*, 1991). Since the characteristic energy scale associated with the last level is $\sim\Delta$, one finds $I\sim\Delta/\phi_c$ for the maximal total current. In the same fashion, using $\chi \propto \triangle/\phi_c^2$, the AB susceptibility is estimated as $\chi \sim |\chi_L|(E_c/\max\{2\pi T, \gamma\}) \le |\chi_L|(E_c\triangle)$ where $\chi_L = -(1/3)\mu_B^2 N(0)$ is the Landau diamagnetic susceptibility and μ_B the Bohr magneton (the exact prefactor can be derived with the help of Eq. (8)')).

Direct Response

We now turn to the evaluation of the response of an isolated disk of radius $L/2\pi$. First, we consider a two-dimensional situation and then generalize it to a slab of thickness $a \ll L/2\pi$. We first concentrate on the contribution of the zero mode-the lowest eigenvalue solution for the Cooperon (Serota *et al.*, 1987). In the absence of magnetic field, the zero mode of the Cooperon is a constant in space $\propto [-i(\varepsilon_1-\varepsilon_2)+\gamma]^{-1}$, whose derivative at the boundary is zero indicating the absence of electronic flux in and out of the sample. In a weak field, the zero eigenvalue is shifted by a small amount which can be evaluated perturbatively. The perturbation theory here (Oh *et al.*, 1991) is different from the traditional stationary state perturbation theory due to boundary condition of zero covariant derivative (Spivak and Khmelnitskii, 1982). The latter condition does not allow the usual expansion of the first-order correction to the eigenstste in terms of the unperturbed eigenstates. Also, in integrations by parts, one can not neglect the surface terms giving extra contributions to the eigenvalues. The result is that the first-order correction to the lowest eigenstate is zero, which should be expected on the basis of time-reversal symmetry argument. The second-order magnetic correction can be expressed in terms of volume integral of the square of the vector potential in a gauge where the vector potential is tangential to the surface of the sample. For a round disk, this would simply be the radial gauge. As a result we arrive at the following expression for the zero mode contribution to the free energy of the canonical ensemble :

$$F-\Omega = \frac{s^2\triangle}{(4\pi)^2}\int\int d\varepsilon_1 d\varepsilon_2 f(\varepsilon_1) f(\varepsilon_2)\,\mathrm{Re}\left[(\varepsilon_1-\varepsilon_2)+i\gamma+i\frac{E_c}{2}\left(\frac{2\phi_D}{\phi_0}\right)^2\right]^{-2}. \quad (9)$$

Eg. (9) allows us to evaluate the magnetic moment and the susceptibility of the disk. As in the AB case, for flux values such that $2\phi_D \le \phi_c$, the r. h. s. of Eq. (9) gives the dominant contribution and can be expanded lineary in flux. By differentiating twice with ϕ_D, magnetic susceptibility is given like

$$\chi \sim |\chi_L| \begin{cases} 4E_c/(2\pi T), & E_c \gg 2\pi T \gg \gamma \\ 12E_c/(\pi^2\gamma), & E_c \gg \gamma \gg 2\pi T \end{cases}. \quad (10)$$

Again, when $\max\{2\pi T, \gamma\} \le \triangle$, the quantity of order $\triangle$ should be substituted for T and γ in Eq. (10). For a slab of thickness $a \ll L$, the 3D density of states yields an extra factor of $\pi^2/(k_F a)^2$ in the r. h. s. of Eq. (10). Significance of the characteristic flux ϕ_c, below which the linear response is valid, is confirmed via the evaluation of the correlation function of the chemical potential Eq. (2) for $E_c \gg 2\pi T \gg \gamma$

$$\begin{aligned}\langle\delta\mu(\delta\phi_D)\delta\mu(0)\rangle - \langle[\delta\mu(0)]^2\rangle &= \frac{\triangle^2 s^2}{\pi^2}\sum_{m=1}^{E_c/2\pi T}\left[\frac{m}{(m+4z)^2}-\frac{1}{m}\right] \\ &= \frac{\triangle^2 s^2}{\pi^2}\left[\frac{2}{z}-\Gamma(z+1)+z\{\Gamma'(z+1)-2\Gamma'(z)\}-C\right]\end{aligned} \quad (11)$$

where $z=(1/2)(2\pi\delta\phi_D/\phi_c)^2$, Γ' is the digamma function, and C is the Euler's constant. It saturates to the log z at $\delta\phi_D \sim \phi_c$. As in the AB case, the zero mode contribution rapidly decays above ϕ_c. Therefore, one has to take into account the contribution to the susceptibility from the free energy of the grand canonical ensemble in Eq. (9). The latter is known to yield the diamagnetic Landau susceptibility which survives in the disorder averaging (Fukuyama, 1971) and stronger field $\phi \geq \phi_0$. The higher modes have to be taken into accounts as well. For these modes, the terms of the order E_c appear in the Cooperon and validity of linear response extends to the flux quantum. Although the contribution of each mode is easily estimated to be order of χ_L, we were unable to evaluate the total contribution of higher modes.

Conclusion

In conclusion, we have shown that the contribution of the zero mode of the Cooperon to the direct average paramagnetic response of a canonical ensemble, represented here by an isolated disordered metallic disk penetrated by the magnetic flux, is identical to that of the AB response. For weak magnetic fields, the total magnetic moment of the system is given by

$$M \sim \mu_B \frac{2\phi}{\phi_0} \begin{cases} E_c/\max\{2\pi T, \gamma\}, & \max\{2\pi T, \gamma\} \geq \Delta \\ E_c/\Delta, & \max\{2\pi T, \gamma\} \leq \Delta \end{cases}, \quad (12)$$

where ϕ is the flux through the metal, ϕ_D, for the direct response or the flux through the hole of the ring, ϕ_{AB}, for the AB response. Eq. (12) can be translated into the following expression for the magnetic susceptibility which is much larger than the Landau susceptibility χ_L:

$$\chi \sim |\chi_L| \begin{cases} E_c/\max\{2\pi T, \gamma\}, & \max\{2\pi T, \gamma\} \geq \Delta \\ E_c/\Delta, & \max\{2\pi T, \gamma\} \leq \Delta \end{cases}, \quad (13)$$

neglecting $\partial^2\Omega/\partial H^2 \propto \exp(-l/L)$ in the AB case (Cheung et al., 1989) and $V^{-1}\partial^2\Omega/\partial H^2 = \chi_L$ in the direct response (Fukuyama, 1971) respectively. Here, l is the mean free path of electrons and H denotes the external magnetic field. We find that the average response of the canonical ensemble is paramagnetic. This is shown via the evaluation of an extra term appearing in the free energy of the canonical ensemble, as compared to the grand canonical ensemble, and corresponding to the contribution of the energy levels closest to the Fermi level. In addition to the evaluation of such a term for the direct response, we have also discovered that the large magnetic susceptibility originates in the so called zero mode, the zero eigenvalue solution for the Cooperon, imposed by the condition of the absence of the current through the sample boundaries. In the AB case, the persistent current reaches its maximal value of $(e\Delta/\hbar)(E_c/\max\{2\pi T, \gamma\})$ at the chracteristic flux of ϕ_c and then rapidly falls off. This pattern repeats itself with periodicity of half the flux quantum. On the other hand, a solid metallic disk in the magnetic field exhibits a much more interesting behavior. For the flux larger than ϕ_c, the zero mode contribution is suppressed. The remaining contributions to the susceptibility are ones due to the higher modes of the Cooperon and the omnipresent Landau diamagnetic susceptibility χ_L, obtained from the free energy of a grand canonical ensemble. As the flux through the disk is increased over half the flux quantum, the contribution of the Cooperon will become suppressed leaving behind only the usual Landau diamagnetic susceptibility.

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〈국문초록〉

Aharonov-Bohm 반응과 직접 반응에서의 상자성

정준 앙상블의 Aharonov-Bohm (AB) 반응에 대한 level statistics를 이용하여, mesoscopic 금속고리에서의 영구전류의 엄밀한 표현식을 구하였다. 그리고 직접반응의 예로 원형 금속판의 불순물-평균한 궤도 상자성 감수율을 계산한 결과, AB 경우와 마찬가지로 이차원에서 상자성 감수율이 $\chi_{para} \sim |\chi_L|(E_c/\max(2\pi T, \gamma))$로 주어졌다.

자기선속이 $\phi_c \sim \phi_0 (E_c/\max(2\pi T, \gamma)^{-1/2}$ 보다 작은 경우, 상자성 감수율은 대 정준 앙상블의 불순물-평균한 반자성 감수율 (Landau 반자성 감수율) 보다 크며 선속이 선속양자의 1/2 정도의 크기를 가질 때 자기감수율은 Landau 반자성 감수율 정도의 크기를 갖는다.