濟州大學校 尖端技術硏究所 論文集, 第12卷, 2號, pp.246-250, 2001. 12
Cheju Nat'l Univ. Res. Inst. Adv. Tech. Jour., Vol.12, No.2, pp.246-250, 2001. 12

Stability analysis of an initially, stably stratified fluid subjected to a step change in temperature

Min Chan Kim* and Sang Baek Lee*

급격하게 온도가 변하는 초기에 안정하게 성층화된 유체층에서의 안정성 해석

김 민 찬* · 이 상 백*

ABSTRACT

The onset of convective instability in an initially quiescent, stably stratified fluid layer between the two horizontal plates is analysed with linear theory. The layer is heated suddenly from below, subjected to a step change in surface temperature. The dimensionless critical time τ_c is to mark the onset of Rayleigh-Benard convection is obtained numerically by using propagation theory. The results show that disturbances manifest themselves around $\tau = 4\tau_c$ in comparison with available experimental data.

Key Words : stability analysis, initially stable fluid layer, propagation theory

I. INTRODUCTION

When a fluid layer confined between two the horizontal plates is heated rapidly from below, Rayleigh-Benard convection can set in at a certain time due to buoyancy forces. In this transient-heating system the important problem is to find the critical time to mark the onset of convective motion.

The system considered here is sketched in Fig. 1. Initially the quiescent fluid layer of depth d is stratified stably with temperature $T = T_i$ at the vertical distance $Z = 0$ and $T = T_u$ ($\geq T_i$). Starting from time $t = 0$, the bottom boundary is heated uniformly at a higher temperature T_b. For small time the base temperature profile of heat conduction will be nonlinear and time-dependent. The important parameters in this thermally developing system are the Rayleigh number Ra ($= g\beta\Delta T d^3/\nu\alpha$), the Prandtl number Pr ($=\nu/\alpha$) and the temperature ratio γ ($=(T_u - T_i)/(T_b - T_u)$). Here g, β, ΔT, ν and α denote the gravitational acceleration, the thermal expansivity, the temperature difference across the boundaries ($= T_b - T_u$), the kinematic viscosity, and the thermal diffusivity, respectively. The object of this study is to find the dimensionless critical time τ_c to mark the onset of convective instability for a given Pr, Ra and γ. Here τ ($=\alpha t/d^2$) denotes the Fourier number. We will employ propagation theory, which is based on the assumption that temperature

* 제주대학교 화학공학과, 첨단기술연구소
Department of Chemical Engineering, Research Institute of Advanced Technology, Cheju Nat'l Univ.

disturbances at $\tau=\tau_c$ would be propagated mainly to the thermal penetration depth of conduction state. The present results will complement Kim et al.'s[1)] work.

II. PROPAGATION THEORY

For the present system the dimensionless basic temperature $\theta_0\,(=(T_0-T_u)/(T_b-T_i))$ of the conduction state can be obtained[1)] :

$$\theta_0=(1-\gamma)(1-z)-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi z)}{n}\exp(-n^2\pi^2\tau) \qquad (1)$$

$$\theta_0=-\gamma(1-\zeta\sqrt{\tau})+\sum_{n=0}^{\infty}\left\{\mathrm{erfc}\left(\frac{n}{\sqrt{\tau}}+\frac{\zeta}{2}\right)-\mathrm{erfc}\left(\frac{n+1}{\sqrt{\tau}}-\frac{\zeta}{2}\right)\right\} \qquad (2)$$

where $z=Z/d$ and $\zeta=z/\sqrt{\tau}$ Here to denotes the basic temperature. Equations (1) and (2) yield the same temperature profile but have the different coordinates.

Under linear theory the perturbed quantities are expressed in terms of the temperature component θ_1 and the vertical velocity component w_1 as

$$\left(\frac{1}{Pr}\frac{\partial}{\partial\tau}+\nabla^2\right)\nabla^2 w_1=\nabla_1^2\theta_1 \qquad (3)$$

$$\frac{\partial\theta_1}{\partial\tau}+Ra\ w_1\frac{\partial\theta_0}{\partial z}=\nabla^2\theta_1 \qquad (4)$$

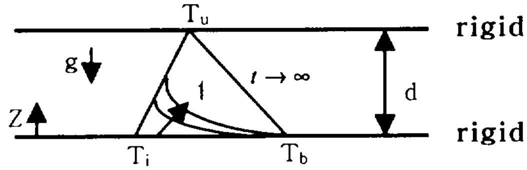

Fig. 1. Temperature profiles in conduction state.

where ∇_1^2 denotes the horizontal Laplacian. Here w_1 has the scale of α/d and θ_1 that of $\alpha\nu/(g\beta d^3)$. The proper boundary conditions are given by

$$w_1=\frac{\partial w_1}{\partial z}=\theta_1=0 \quad \text{at} \quad z=0 \text{ and } 1 \qquad (5)$$

which represent no slip and isothermal heating on the boundaries.

For small τ the dimensionless amplitude functions of disturbances are expressed under the normal mode analysis, based on the balance between the viscous and buoyant forces in the z−component of motion [1] :

$$[w_1(\tau,x,y,z),\theta_1(\tau,x,y,z)]=[\tau w^*(\zeta),\theta^*(\zeta)]\exp[i(a_x x+a_y y)] \qquad (6)$$

where i is the imaginary number and a_x and a_y denotes the wavenumbers. With $\gamma=0$, substituting Eqs. (6) into (3) and (4) yields

$$(D^2-a^{*2})^2 w^*=a^{*2}\theta^*-\frac{1}{Pr}\left[\frac{\zeta}{2}D^3w^*-\frac{\zeta}{2}a^{*2}Dw^*+a^{*2}w^*\right] \qquad (7)$$

$$(D^2-a^{*2})\theta^*=-\frac{\zeta}{2}D\theta^*+Ra^*w^*D\theta_0 \qquad (8)$$

where $D=d/d\zeta$, $a^*=\tau^{1/2}a$, $a=\sqrt{a_x^2+a_y^2}$, and $Ra^*=\tau^{3/2}Ra$. Here a^* and Ra^* are assumed to be eigenvalues. This makes it possible to produce the above self−similar equations including $\theta_0(=\mathrm{erfc}(\zeta/2))$ from Eq. (2) as a function of $\zeta(=z/\sqrt{\tau})$ only because the upper boundary is replaced by $\zeta(=1/\sqrt{\tau})\to\infty$ for small τ. Now, the minimum Ra^*−value are obtained numerically. From characteristic values the critical time τ_c and the critical horizontal wavenumber a_c are obtained for a given Ra. Also, the critical Rayleigh number

Ra_c may be obtained at each τ_c. This means that τ may be fixed as τ_c but ζ varies in the stability equations.

The above procedure is extended to the case of $\gamma > 0$ and also to that of large τ. As shown in Eq. (2), the resulting equations are not self-similar. But we fix τ as τ_c in Eqs. (5), (7) and (8). Now, for a given τ_c, γ and Pr the minimum Ra-value Ra_c is found. Therefore the propagation theory introduced above is a kind of relaxed frozen-time model. and an accurate estimation of the spatially varying thermal conductivity is necessary in many thermal management systems. Also, this kind of problem may be encountered in geological waste disposal and also have applications in petroleum field and aquifer analysis.

III. RESULTS AND DISCUSSION

The dimensionless critical time τ_c to mark the onset of convective instability has been obtained by propagation theory. Figure 2 shows that the system becomes more stable as γ increases and as Pr decreases. For

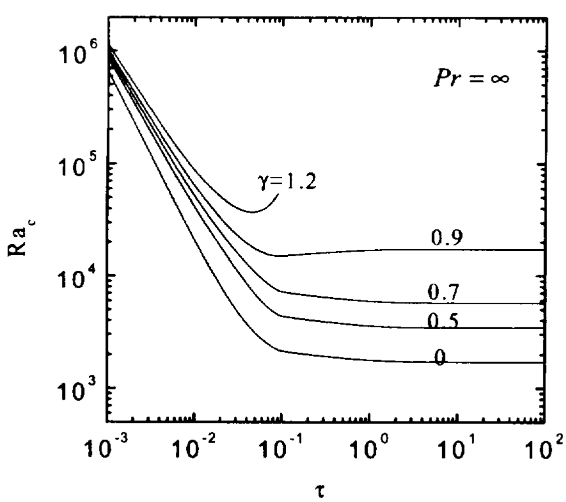

Fig. 2. Effect of the temperature ratio γ on the critical time τ_c.

large τ_c, Ra_c approaches the well-known value of $1708/(1-\gamma)$, independently of Pr. For large γ-value both the multiple-cell patterns and the behavior of subcritical state are exhibited, producing the minimum Ra_c-value in the plot of Ra_c vs. τ. For $\gamma \geq 1$ the instabilities will disappear with increasing time and the system becomes unconditionally stable as $\tau \rightarrow \infty$. For deep-pool systems of small τ_c the stability criteria are summarized in Kim et al.'s[1] work. Ueda et al.[2] conducted experiments of $Ra = 9000 \sim 17000$, $\gamma = 0.73 \sim 1.67$ and $Pr = 8800$ obtained the characteristic time τ_m to mark the detection of manifest convection. Comparison with the present predictions yields the relation of $\tau_m \cong 4\tau_c$ for $\gamma Ra^{-1/3} < 0.03$, as shown in Fig. 3. Here their predictions from the amplification theory are also compared. The significant deviation of $4\tau_c$-values from the last two data may be caused by $\gamma > 1$ in experiments.

$$\tau_c = 7.53[1 + (\frac{0.804}{Pr})^{3/4}]^{8/9} Ra^{-2/3} \qquad (9)$$

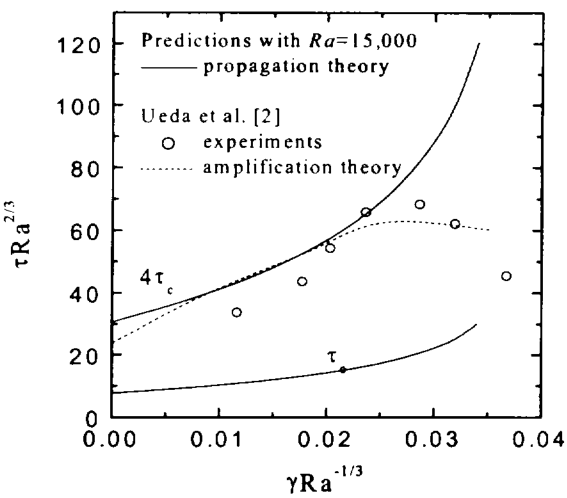

Fig. 3. Comparison of present predictions with Ueda et al.'s[2] results for a given Ra. With $\gamma = 0$ the present predictions fit would manifest itself at

for $\tau_c < 0.01$ within the error bound of 5%. It has been reported [3),4)] that convection

$$\tau = \tau_m \text{ with the relation of } \tau_m \cong 4\tau_c \qquad (10)$$

because incipient disturbances at $\tau = \tau_c$ must grow with time. The τ_c predicted by the amplification theory[1),5)] and the stochastic model[6)] also yield the above relation when τ_c-values is obtained from Eq. (9). Patrick and Wragg[7)] measured the individual mass transfer coefficient with time in electroplating systems, which correspond to those of $Pr > 2{,}000$. Figure 4 shows that their undershoot times are well represented by the relation (10). The undershoot time indicates the minimum Nusselt number Nu in the plot of Nu vs. τ.

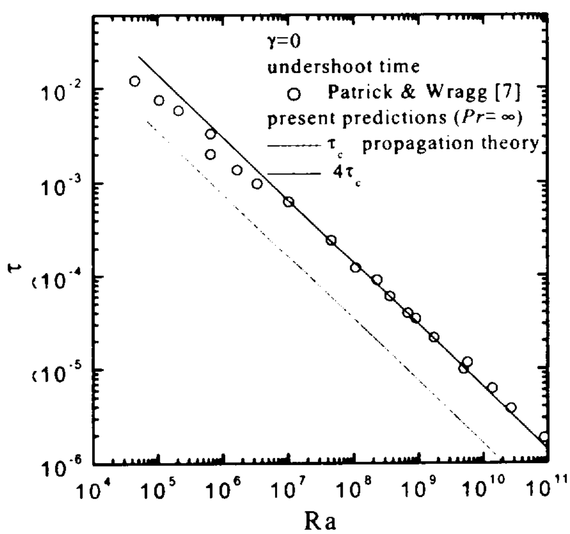

Fig. 4. Comparison of present predictions with experimental data of Patrick and Wragg[7)] for a given Ra.

IV. CONCLUSION

Even though propagation theory is a rather simple model, it seems that the resulting stability criteria are consistent with experimental measurements. The present results show that the infinitesimal disturbance sets in at $\tau = \tau_c$ and for large Pr-systems it grows until detected around $\tau = 4\tau_c$. It is interesting that the propagation theory can be applied to the stability analysis of diffusive systems without the loss of generality.

요 약

두 수평한 사이에서 초기에 안정하게 성층화된 정지상태의 유체층에서 급작스럼 가열에 의한 대류 불안정성 발생을 선형 이론을 도입하여 해석하였다. Rayleigh-Benard 대류의 발생을 나타내는 임계시간 τ_c를 전파이론을 사용하여 수치적으로 구하였다. 기존의 실험결과와 비교하여 볼 때, 교란들은 $\tau = 4\tau_c$ 에서 가시화 되는 것으로 보여진다

REFERENCES

1) M.C. Kim, K.H. Choi and C.K. Choi, 1999, Onset of thermal convection in an initially stably stratified fluid layer, Int. J. Heat Mass Transfer, Vol. 42, pp. 4253-4258.
2) H. Ueda, S. Komori, T. Miyazaki and H. Ozoe, 1984, Time-dependent thermal convection in a stably stratified fluid layer heated from below, Phys. Fluids, Vol. 27, pp. 2617-2623.
3) Kang, K.H., Choi, C.K. and Hwang, I.G., 2000, "Onset of solutal Marangoni convection in a suddenly desorbing liquid layer," AIChE J., Vol. 46, pp. 15-23.

4) Foster, T.D., 1969, "Onset of manifest convection in a layer of fluid with a time-dependent surface temperature," Phys. Fluid, 12:2482-2487.
5) Gresho, P.M. and Sani, R.L., 1971, "The stability of a fluid layer subjected to a step change in temperature: transient vs. frozen time analysis," Int. J. Heat Mass Transfer, 14:207-221.
6) Jhaveri, B.S. and Homsy, G.M., 1982, "The onset of convection in fluid layers heated rapidly in a time-dependent manner," J. Fluid Mech., 114: 251-260.
7) Patrick, M.A. and Wragg, A.A., 1975, "Optical and electrochemical studies of transient free convection mass transfer at horizontal surfaces" Int. J. Heat Mass Transfer, 18:1397-1405.