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THE UNIQUENESS OF SOLUTIONS FOR DIFFERENTIAL EQUATIONS ON THE WIENER SPACE

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ABSTRACT. We consider a differential equation on the Wiener space. A capacity is defined by Ornstein-Uhlenbeck operator. We show the uniqueness of solutions for some differential equation on the Wiener space using the flow property and quasi sure convergences.

1. Introduction

The solutions for a differential equation generating by a vector field on $\mathbb{R}^n$ exist for all $x \in \mathbb{R}^n$ and satisfy the flow property for initial values clearly [7]. The quasi sure existence of solutions on the Wiener space, e.g., the space of $\mathbb{R}^n$-valued continuous paths starting at 0, that is the existence of solutions for all initial values except in a set of (r,p)-capacity 0 for all $r \geq 0$ and $p > 0$, has proved [7], and the solutions have a flow property quasi surely for initial values [8]. Here the capacity is associated with the Ornstein-Uhlenbeck operator.

Cruzeiro proved the almost sure existence of solutions having flow property for some differential equation on $\mathbb{R}^n$ [1]. She proved also the uniqueness of solutions for the differential equation on Wiener space using almost sure flow property [2]. In this paper, we show the uniqueness of solutions. But since the uniqueness can be proved by the same way, we only prove the almost sure flow property by different method to that of Cruzeiro using a finite dimensional approximations and the quasi sure convergences proved in [8].

We use the notion of Sobolev spaces of Banach valued functions due to Shigekawa [6]. To be precise, let (X, H, μ) be an abstract Wiener space and A be a vector field on X which is, by definition, a mapping from X into H smooth in the sense of Malliavin. Then under some conditions for A, the solutions $V_t(x)$ of the following differential equation (1.1) exist for all $t \in \mathbb{R}$, quasi everywhere x (q.e. x in abbreviation) [7].

$$(1.1) \qquad \begin{cases} (dV_t/dt)(x) = A(V_t(x) + x), \\ V_0(x) = 0. \end{cases}$$

The problem was first treated by Cruzeiro [1] and she established for a class of vector fields the existence and the uniqueness of solutions $U_t(x)$ for μ-almost every (μ-a.e.) $x \in \mathbb{R}^n$ which satisfy the almost everywhere flow property: for every $t \in \mathbb{R}$, $(U_t)_*\mu$ is absolutely continuous with respect to μ and satisfies

$$(1.2) \qquad U_t \circ U_s(x) = U_{t+s}(x) \quad \text{for all} \quad t, s \in \mathbb{R}, \quad \mu - \text{a.e. } x \in \mathbb{R}^n.$$

The flow property has proved for μ − a.e. $x \in X$ under a little stronger condition. Then using the flow property, Cruzeiro proved the uniqueness of solutions for differential equation on Wiener space [2]. We will show the above solutions $V_t(x)$ of (1.1) satisfy the flow property (1.2) almost surely by different method to that of Cruzeiro in the following manner :

$$U_t \circ U_s = U_{t+s}, \quad \text{where} \quad U_t(x) = V_t(x) + x, \quad \text{for all} \quad t, s \in \mathbb{R}.$$

We assume the hypothesis of Theorem 2.3 in this paper. We take a sequence A_n converging to A such that A_n depends only on finite number of coordinates and takes values in finite dimensional subspace of H. Denoting a solution for A_n by $V_t^{(n)}(x) \equiv U_t^{(n)}(x) - x$. We proved $V_t^{(n)}$ converges quasi surely and the limit satisfies (1.1) [7]. Since the equation (1.1) can be rewrited as

$$U_t^{(n)}(x) = x + \int_0^t A_n(U_s^{(n)}(x))ds, \quad \text{for} \quad U_t^{(n)},$$

and

$$U_t^{(n)}(U_s^{(n)}(x)) = U_s^{(n)}(x) + \int_0^t A_n(U_r^{(n)}(U_s^{(n)}(x)))dr, \quad \text{for} \quad U_t^{(n)}(U_s^{(n)}(x)),$$

for the almost sure flow property, we prove $\int_0^t A_n(U_r^{(n)}(U_s^{(n)}(x)))dr$ converges to
$\int_0^t A(U_r(U_s(x)))dr$ almost sure in the section 3 using some finite dimensional approximations prepared in the section 2 and quasi sure convergences proved in [7].

2. Existence of solutions and approximations

We denote by μ the standard Gaussian measure on $\mathbb{R}^n$. By a vector field on $\mathbb{R}^n$, we mean a Borel measurable function $B : \mathbb{R}^n \ni x \mapsto B(x) \in \mathbb{R}^n$.

Definition 2.1. Let $A_n : \mathbb{R}^n \to \mathbb{R}^n$ be a vector field in $L^2(\mu)$. The *divergence* of A_n with respect to μ (if it exists) is the element $\delta_\mu A_n \in L^2(\mu)$ satisfying

$$\int_{\mathbb{R}^n} \delta_\mu A_n \cdot u d\mu = \int_{\mathbb{R}^n} (A_n|\nabla u) d\mu$$

for all $u \in W_1^2(\mu)$ where $W_1^2(\mu)$ be the space $\{u \in L^2(\mu) : ||\nabla u|| \in L^2(\mu)\}$ with the norm $||u||_{L^2} + ||\nabla u||_{L^2}$.

For the existence of the flow $U_t^{(A_n)}(x)$, the following theorem has been proved.

Theorem 2.2. (Yun [7], Theorem 3.5) Suppose that $A_n \in C^\infty$ and
(i) $\forall m = 0, 1, 2, \cdots, \forall \lambda > 0$, $\int_{\mathbb{R}^n} \exp(\lambda \cdot ||\nabla^m A_n(x)||) d\mu(x) < +\infty$,
(ii) $\forall \lambda > 0$, $\int_{\mathbb{R}^n} \exp(\lambda|\delta_\mu A_n(x)|) d\mu(x) < +\infty$.
Then a solution of the below system of differential equations (2.1), (2.2), (2.3) and (2.4) below exists for all $t \in \mathbb{R}$ starting for all $x \in \mathbb{R}^n$.

$$\frac{d}{dt} V_t(x) = A_n(V_t(x) + x), \tag{2.1}$$

$$\frac{d}{dt} \nabla V_t(x) = \nabla A_n(V_t(x) + x) \cdot \nabla V_t(x) + \nabla A_n(V_t(x) + x), \tag{2.2}$$

$$(2.3)\qquad \frac{d}{dt}LV_t(x) = LA_n(V_t(x)+x) + \nabla A_n(V_t(x)+x)\cdot LV_t(x)$$
$$+\sum_i \nabla^2_{jk}A_n(V_t(x)+x)\cdot \partial_i V_t^j(x)\cdot \partial_i V_t^k(x)$$
$$+2\sum_i \partial_i \nabla_j A_n(V_t(x)+x)\cdot \partial_i V_t^j(x),$$
$$V_0(x) = \nabla V_0(x) = LV_0(x) = 0.$$

More generally, in addition to (2.1)$\sim$ (2.3), we consider the following system of differential equations to be satisfied by

$$[L^m\nabla^n V_t(x) : m = 0,1,\cdots,N,\quad n=0,1,\cdots,2N,\quad 2m+n\le 2N]:$$

$$(2.4)\quad \frac{d}{dt}L^m\nabla^n V_t = \nabla A_n\cdot L^m\nabla^n V_t$$
$$+E^{m,n}(L^i\nabla^j A_n, L^l\nabla^r V_t : i=0,1,\cdots,m,$$
$$j=0,1,\cdots,n,\quad l=0,1,\cdots,m-1,$$
$$r=0,1,\cdots,n,\quad 2i+j\le 2m+n,$$
$$2l+r\le 2(m-1)+n),$$
$$L^m\nabla^n V_0(x)=0,\quad m=0,1,\cdots,k,\quad n=0,1,\cdots,2k,$$
$$2m+n\le 2k,\quad k=2,3,\cdots,N,$$

where $E^{m,n}$ is some polynomial which can be obtained successively (see the proof of Theorem 5.5, Yun [7]).

Let (X,H,μ) be an abstract Wiener space introduced by L. Gross [3] where
(i) X is a real, separable Banach space with the norm $||\cdot||$,
(ii) H is a real, separable Hilbert space densely included in X with the inner product $\langle x,y\rangle_H$,
and

(iii) μ is the standard Gaussian measure, i.e., the Borel probability measure on X such that

$$\int_X \exp\{i(h,x)\}\mu(dx) = \exp(-\frac{1}{2}\langle h,h\rangle_H)$$

where $h \in X^* \subset H$ and $(\ ,\ )$ is a natural pairing of X^* and X. Note that $||x|| \leq k|x|_H = k\sqrt{\langle x,x\rangle_H}$, $x \in H$ for some constant $k > 0$ so that the inclusion map $i : H \to X$ is continuous. Hence we have $X^* \subset H^* = H$ and we regard X^* as a subset of H.

The Sobolev space W_r^p on $X(1 \leq p < \infty)$ defined in [5] is the space of functions ϕ on X such that $||\phi||_{L^p(\mu)}$ and for all $1 \leq i \leq r$, $||\nabla^i\phi||_{L^p(\mu)} < \infty$ with a norm

$$||\phi||_{r,p} = ||\phi||_{L^p(\mu)} + \sum_{i=1}^{r} ||\nabla^i\phi||_{L^p(\mu;\mathcal{L}^i_{(2)}(H;G))}.$$

We denote by W_∞^p the space $\cap_r W_r^p$. Let f be a vector field on X that is a mapping from X into H. If $f \in L^2(X;H)$, we denote by δf the divergence, i.e., δ is the adjoint operator of ∇ for the Wiener measure, more precisely, δf is the element of $L^2(X)$ (if it exists) satisfying

$$\int_X \delta f\theta d\mu = \int_X \langle f|\nabla\theta\rangle_H d\mu$$

for all function $\theta \in W_1^2(X)$.

For $[0,\infty]$-valued lower semicontinuous (l.s.c. in abbreviation) function h, define $C_{r,p}(h)$ by

$$C_{r,p}(h) := \inf\{||u||^p_{r,p}; u \in W_r^p(B), u \geq h, \mu\text{-a.e.}\}$$

and for an arbitrary $[-\infty,\infty]$-valued function f (not assumed to be μ-measurable),

$$C_{r,p}(f) := \inf\{C_{r,p}(h); h \text{ is l.s.c. and } h(x) \geq |f(x)|, \forall x \in X\}.$$

For a set G, we define

$$C_{r,p}(G) = C_{r,p}(1_G).$$

Here 1_G denotes the indicator function of G.

We say that a property holds quasi-everywhere (q.e. in abbreviation) if it holds except on a set of capacity 0 for all r, p. We note that the following property holds for Sobolev spaces on an abstract Wiener space.

$$W_r^p(B) \cap C_b(X \to B) \text{ is dense in } W_r^p(B) \text{ and } 1 \in W_r^p(B).$$

Then it has been proved by I. Shigekawa that any $v \in W_r^p(B)$ admits a quasi-continuous modification and denoting it by $\tilde{v}$, it holds that

$$C_{r,p}(||\tilde{v}||_B) \leq ||v||_{r,p}^p$$

and the Chebyshev type inequality holds

$$C_{r,p}(||\tilde{v}|| \geq \lambda) \leq \frac{1}{\lambda^p}||v||_{r,p}^p.$$

Using the above facts, we have the following theorem.

Theorem 2.3. (Yun [7], Theorem 5.5) If A is a vector field on X satisfying
(i) $A \in W_\infty^\infty(X; H)$ and $\forall\lambda > 0$, $\int_X \exp(\lambda||A(x)||)d\mu(x) < +\infty$,
(ii) $\forall\lambda > 0$, $\forall n = 1, 2, \cdots$, $\int_X \exp(\lambda||\nabla^n A(x)||)d\mu(x) < +\infty$ and
(iii) $\forall\lambda > 0$, $\int_X \exp(\lambda|\delta A(x)|)d\mu(x) < +\infty$,
then $V_t(x)$ exists for all $t \in \mathbb{R}$, q.e.x satisfying the differential equation (1.1).

We can easily prove the following general facts for flows on $\mathbb{R}^n$ (Cruzeiro [1], Lemma 3.1.1).

Lemma 2.4. Let A and B be C^3-vector fields on $\mathbb{R}^n$ generating the global flows U_t^A and U_t^B. Let $h = A - B$. Then the following $(i) \sim (iv)$ hold.

(i) $$U_t^A = U_t^B \circ U_0^{t,Z},$$

where $U_s^{t,Z}$ is defined by

$$\left(\frac{\partial U_s^{t,Z}}{\partial t}\right)(x) = Z^t(U_s^{t,Z}(x)), \quad U_s^s(x) = x,$$

and Z is given by

$$Z^t(y) = (U_{-t}^B)'(y)h(U_t^B(y)).$$

(ii)
$$U_t^A = (U_{-t}^{0,Z}) \circ U_t^B.$$

(iii)
$$U_t^A(x) - U_t^B(x) = -\int_0^{-t} (U_{t+\xi}^B)'(U_\xi^{0,Z}(y)) \cdot h(U_{-t-\xi}^B(U_\xi^{0,Z}(y)))d\xi,$$

where $y = U_t^B(x)$.

(iv)
$$\begin{aligned}\frac{d}{d\lambda}U_\lambda^{0,Z}(x) = &-(U_0^B)'(U_\lambda^{0,Z}(x)) \cdot (A-B)(U_\lambda^{0,Z}(x)) \\ &- \int_0^\lambda \left[\frac{d}{d\lambda}(U_{-\lambda+\xi}^B)'(y) \cdot (A-B)(U_{\lambda-\xi}^B)(y)\right. \\ &\left. + (U_{-\lambda+\xi}^B)'(y) \cdot \frac{d}{d\lambda}((A-B)(U_{\lambda-\xi}^B(y)))\right] d\xi,\end{aligned}$$

where $y = U_\xi^{0,Z}(x)$.

In the following, we denote by $U_t^{(n)} := U_t^{A_n}, U_t^{(m)} := U_t^{A_m}, U_t := U_t^A, \cdots$. By Theorem 2.2, the field A_n has the flow $U_t^{A_n} \equiv U_t^{(n)}$ which can be defined for all $t \in \mathbb{R}$ starting for all $x \in \mathbb{R}^n$. Further we have, by the formula of change of variables (Theorem 2.2),

(2.5)
$$\frac{d(U_t^{(n)})_*\mu}{d\mu}(x) = \exp\left(\int_0^t \delta A_n(U_{-\xi}^{(n)}(x))d\xi\right) = k_t^n(x), \quad x \in \mathbb{R}^n.$$

This flow is a transformation of $\mathbb{R}^n$. We must modify the transformation on X and at the same time it does not change the formula (2.5).

We can set, for $x \in X$,

$$x = y + z, \quad z \in H_n.$$

If $U_t^{(n)}$ is the flow for A_n defined on $\mathbb{R}^n$, set

$$V_t^{(n)}(x) = U_t^{(n)}(z) + y - x = U_t^{(n)}(z) - z.$$

Then

$$\frac{V_t^{(n)}(x)}{dt} = A_n(V_t^{(n)}(x) + x) \quad \text{and} \quad V_0^{(n)}(x) = 0.$$

If ϕ is a function defined on H_n, then we have $\phi(V_t^{(n)}(x) + x) = \phi(U_t^{(n)}(x))$.

On the other hand, the measure μ can be decomposed as $\mu = \mu_n \otimes \nu_n$ with ν_n defined on $H_n^{\perp}$. Then we have

$$\begin{aligned}
&\int_X \psi(x) d(V_t^{(n)}(\cdot) + \cdot)_* \mu(x) \\
&\qquad = \int_{H_n^{\perp}} \left(\int_{H_n} \psi(V_t^{(n)}(y+z) + y + z) d\mu_n(z) \right) d\nu_n(y) \\
&\qquad = \int_X \psi(x) \cdot \exp\left(\int_0^t \delta A_n(U_{-\xi}^{(n)}(x)) d\xi \right) d\mu(x).
\end{aligned}$$

Thus we have the following

$$\text{(2.6)} \qquad \frac{d(V_t^{(n)}(\cdot) + \cdot)_* \mu}{d\mu}(x) = \exp\left(\int_0^t \delta A_n(V_{-\xi}^{(n)}(x) + x) d\xi \right) = k_t^n(x).$$

We have proved the following equation (Yun [7], Lemma 5.4) :

$$\lim_{n,l \to \infty} \left[\sup_{r \in (0,t)} \int_X ||\nabla^m A_n(V_r^{(n)}(x) + x) - \nabla^m A^{(n)}(V_r^{(l)}(x) + x)||^p d\mu(x) \right] = 0,$$

for all $s, t \in \mathbb{R}$ $(t > 0,\ p \geq 1)$ and $m = 1, 2, \cdots$. Using the above lemma, we can prove that

$$\text{(2.7)} \quad \lim_{n \to \infty} \int_0^t \int_X ||\nabla^m A_n(V_r^{(n)}(x) + x) - \nabla^m A_n(V_r(x) + x)||^p d\mu(x) dr = 0.$$

Lemma 2.5. For all $s, t \in \mathbb{R}$ and $p \geq 1$,

$$\lim_{n\to\infty} \int_0^t \int_X ||A_n(V_r^{(n)}(x) + x) - A(V_r(x) + x)||^p d\mu(x) dr = 0.$$

Proof. We have

$$\begin{aligned}
&\left(\int_0^t \int_X ||A_n(V_r^{(n)}(x) + x) - A(V_r(x) + x)||^p d\mu(x) dr\right)^{1/p} \\
&\qquad \leq \left(\int_0^t \int_X ||A_n(V_r^{(n)}(x) + x) - A_n(V_r(x) + x)||^p d\mu dr\right)^{1/p} \\
&\qquad\quad + \left(\int_0^t \int_X ||A_n(V_r(x) + x) - A(V_r(x) + x)||^p d\mu dr\right)^{1/p}.
\end{aligned}$$

By (2.7), the first part of the right hand side tend to 0 as $n \to \infty$. Since the second part is calculated by

$$\begin{aligned}
&\lim_{n\to\infty} \int_0^t \int_X ||A_n(V_r(x) + x) - A(V_r(x) + x)||^p d\mu dr \\
&\qquad \leq \int_0^t ||k_r||_{L^p} \cdot ||A_n - A||_{L^{pq}}^p dr \\
&\qquad \leq C(M) \cdot ||A_n - A||_{L^{pq}}^p \to 0 \quad \text{as} \quad n \to \infty \quad \text{(by Theorem 2.2)},
\end{aligned}$$

the proof is complete. □

3. Almost sure flow property on Wiener space

By Theorem 2.3, the solutions $U_t(x) = V_t(x) + x$ satisfying (1.1) exist q.e. x which is a transformation on the Wiener space (X, H, μ). In this section we prove the almost sure flow property of U_t, that is $U_t \circ U_s = U_{s+t}$, by using the approximations on $\mathbb{R}^n$ and the quasi sure convergences. To be precise, let $U_t^{(n)}$ be the solutions on $\mathbb{R}^n$ satisfying the flow property $U_t^{(n)} \circ U_s^{(n)} = U_{t+s}^{(n)}$

converges to U_{t+s}, q.e. x (see the proof of Theorem 2.3), we have to prove that $U_t^{(n)} \circ U_s^{(n)}$ converges to $U_t \circ U_s$, a.e. x as $n \to \infty$. Note that

$$U_t^{(n)}(U_s^{(n)}(x)) = U_s^{(n)}(x) + \int_0^t A_n(U_r^{(n)}(U_s^{(n)}(x)))dr$$

and

$$U_t(U_s(x)) = U_s(x) + \int_0^t A(U_r(U_s(x)))dr.$$

Since $U_s^{(n)}(x) \to U_s(x)$, q.e. x as $n \to \infty$, there is nothing to prove if we prove that

$$\int_0^t A_n(U_r^{(n)}(U_s^{(n)}(x)))dr \longrightarrow \int_0^t A(U_r(U_s(x)))dr, \quad \text{a.e. } x.$$

Preparing some lemmas, we prove the above convergence in the Theorem 3.3.

Lemma 3.1. For all $s, t \in \mathbb{R}$ $(t > 0)$,

$$\text{(3.1)} \quad \lim_{n,l\to\infty}\left[\sup_{r\in(0,t)} \int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A_n(U_r^{(n)}(U_s^{(l)}(x)))||d\mu\right] = 0.$$

Proof. Using Lemma 2.4 and formula (2.6), we can prove the equation (3.1) (see the proof of Lemma 5.1, Yun [7]). □

Lemma 3.2. For all $s, t \in \mathbb{R}$ $(t > 0)$,

$$\text{(3.2)} \quad \lim_{n\to\infty}\int_0^t \int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A_n(U_r^{(n)}(U_s(x)))||d\mu(x)dr = 0.$$

Proof. Taking subsequence $\{n_j\} \subset \{n\}$ with $U_s^{(n_j)}(x) \to U_s(x)$ a.e. x, then $A_n(U_r^{(n)}(U_s^{(n_j)}(x))) \to A_n(U_r^{(n)}(U_s(x)))$, a.e. x as $j \to \infty$. Since

$$\sup_{n,j}\int_0^t\int_X ||A_n(U_r^{(n)}(U_s^{(n_j)}(x)))||^p d\mu(x)dr$$
$$\leq \sup_{n,j}\int_0^t ||k_s^{n_j}||_{L^2}\cdot ||k_r^n||_{L^4}^{1/2}\cdot ||A_n||_{L^{4p}}^p dr$$
$$< \infty, \quad \forall p\geq 1,$$

we have $A_n(U_r^{(n)}(U_s^{(n_j)}(x))) \to A_n(U_r^{(n)}(U_s(x)))$ in $L^2([0,t]\times X;X)$ as $j\to\infty$, i.e.,

$$\text{(3.3)}\quad \lim_{j\to\infty}\int_0^t\int_X ||A_n(U_r^{(n)}(U_s^{(n_j)}(x))) - A_n(U_r^{(n)}(U_s(x)))||^2 d\mu(x)dr = 0.$$

Note that

$$\int_0^t\int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A_n(U_r^{(n)}(U_s(x)))||_{(n)} d\mu(x)dr$$
$$\leq \int_0^t\int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A_n(U_r^{(n)}(U_s^{(n_j)}(x)))||_{(n,j)} d\mu(x)dr$$
$$+\int_0^t\int_X ||A_n(U_r^{(n)}(U_s^{(n_j)}(x))) - A_n(U_r^{(n)}(U_s(x)))||_{(n,j)} d\mu(x)dr.$$

By the formula (3.3),

$$\lim_{j\to\infty}\int_0^t\int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A_n(U_r^{(n)}(U_s(x)))||_{(n)} d\mu(x)dr$$
$$\leq \lim_{j\to\infty}\int_0^t\int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A_n(U_r^{(n)}(U_s^{(n_j)}(x)))||_{(n,j)} d\mu(x)dr.$$

Since

$$\overline{\lim_{n\to\infty}} \int_0^t \int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A_n(U_r^{(n)}(U_s(x)))||_{(n)} d\mu(x) dr$$
$$\leq \lim_{n\to\infty} \lim_{j\to\infty} \int_0^t \int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A_n(U_r^{(n)}(U_s^{(n_j)}(x)))||_{(n,j)} d\mu(x) dr,$$

we have the equation (3.2) by Lemma 3.1. □

Theorem 3.3. $V_t(x) = U_t(x) - x$ has the flow property for μ − a.e. $x \in X$.

Proof. By Lemma 3.1 and Lemma 3.2, we only have to prove that for all $s, t \in \mathbb{R}$ $(t > 0)$,

$$\lim_{n\to\infty} \int_0^t \int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A(U_r(U_s(x)))|| d\mu(x) dr = 0.$$

Note that

$$\int_0^t \int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A(U_r(U_s(x)))|| d\mu(x) dr$$
$$\leq \int_0^t \int_X ||A_n(U_r^{(n)}(U_s^{(n)}(x))) - A_n(U_r^{(n)}(U_s(x)))|| d\mu(x) dr$$
$$+ \int_0^t \int_X ||A_n(U_r^{(n)}(U_s(x))) - A(U_r(U_s(x)))|| d\mu(x) dr.$$

By Lemma 3.2, the first integral of right hand side tends to 0 as $n \to \infty$ and the second integral is calculated by

$$\int_0^t \int_X ||A_n(U_r^{(n)}(U_s(x))) - A(U_r(U_s(x)))|| d\mu(x) dr$$
$$\leq \int_0^t ||k_s||_{L^2} \cdot \left(\int_X ||A_n(U_r^{(n)}(x)) - A(U_r(x))||^2 d\mu(x) \right)^{1/2} dr$$

and

$$\int_X ||A_n(U_r^{(n)}(x)) - A(U_r(x))||^2 d\mu(x)$$
$$\leq 2\Big(\int_X ||A_n(U_r^{(n)}(x)) - A_n(U_r(x))||^2 d\mu(x)$$
$$+ \int_X ||A_n(U_r(x)) - A(U_r(x))||^2 d\mu(x)\Big).$$

By (2.7), Lemma 2.5 and

$$\int_X ||A_n(U_r(x)) - A(U_r(x))||^2 d\mu(x)$$
$$\leq ||k_r||_{L^2} \cdot \Big(\int_X ||A_n - A||^4 d\mu\Big)^{1/2} \to 0 \quad \text{as} \quad n \to \infty,$$

the proof is complete. □

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