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Phase Shift Analysis for ^{12}C Ion Elastic Scattering at $E_{lab}=300$ MeV

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$E_{lab}=300$ MeV에서 ^{12}C 이온의 탄성산란에 대한 위상이동량 분석

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Summary

A phase shift analysis based on the McIntyre parametrization of S-matrix is presented. This method has been applied successfully to the elastic scatterings of $^{12}C+^{40}Ca$ and $^{12}C+^{90}Zr$ system at $E_{lab}=300$ MeV.

Introduction

Semiclassical methods in the description of heavy ion scattering phenomena are useful approximation techniques when the wavelength associated with the relative motion of the center of mass is very small compared to some typical interaction distance. The widely used method for the analysis of the elastic scattering data is the WKB approximation (Donnelly et al., 1974; Landowne et al., 1976). The alternative methods of analysis are based on the use of the conveniently parametrized scattering matrix, from which the semiclassical expressions for the scattering amplitude have been derived (Frahn, 1985; McIntyre et al., 1960).

The elastic scattering data of ^{12}C ions at intermediate and high energies have been analyzed successfully in the framework of the McIntyre parametrization (Mermaz, 1985; Mermaz et al., 1986). A new representation of the refractive and diffractive parts of the elastic scattering amplitude has been derived and applied to the elastic scattering of $^{12}C+^{12}C$ at $E_{lab}=260$ MeV (Pato et al., 1988). Cha and Kim (1990) have extended the McIntyre parametrization method to the semiclassical case and have applied it successfully to the so-called Fresnel pattern $^{12}C+^{90}Zr$ and $^{12}C+^{208}Pb$ elastic scattering data at $E_{lab}/A=35$ MeV/nucleon. And the angu-

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lar distributions and deflection functions for elastic scatterings of $E_{lab}=210$ MeV ^{6}Li, 216.6 MeV ^{16}O and 222 MeV ^{30}Si beams on ^{208}Pb targets are investigated in terms of McIntyre parametrization and these results are compared with those of the Frahn's generalized Fresnel model (Cha et al., 1990). Sahm et al. have observed the elastic scattering angular distributions for ^{12}C on ^{12}C, ^{40}Ca, ^{90}Zr and ^{208}Pb at $E_{lab}=300$ MeV and have analyzed these data using optical model fits with Woods-Saxon potentials.

In this paper, we present a semiclassical phase shift analysis of elastic scattering data for 300 MeV ^{12}C beams on ^{40}Ca and ^{90}Zr target nuclei based on the McIntyre parametrization of scattering matrix. In Sec. Ⅱ, we present the semiclassical scattering amplitude. Results and conclusions are presented in Sec. Ⅲ.

Semiclassical Scattering Amplitude

The elastic scattering amplitude for spin-zero particle via Coulomb and short-range central forces is given by

$$f(\theta)=f_R(\theta)+\frac{1}{ik}\sum_{l=0}^{\infty}\left(l+\frac{1}{2}\right)\exp(2i\sigma_l)(S_l^N-1)P_l(\cos\theta). \qquad (1)$$

Here $f_R(\theta)$ is the usual Rutherford scattering amplitude, $\sigma_l=\arg\Gamma(l+1+i\eta)$ the Coulomb phase shift and S_l^N denotes the nuclear scattering matrix.

In this work, we use the McIntyre parametrization of the S-matrix (McIntyre et al., 1960). The McIntyre parametrization is expressed for the phase shift of a nuculear S-matrix, S_l^N, as

$$\delta_l=\mu\{1+\exp[(l-\Lambda_1)/\Delta_1]\}^{-1}, \qquad (2)$$

and for the modulus as

$$A_l=\{1+\exp[(\Lambda_2-l)/\Delta_2]\}^{-1}. \qquad (3)$$

From the above two equations, there are five adjustable parameters available for fitting the cross section data : Λ_1, Λ_2, Δ_1, Δ_2 and μ. The two grazing angular momenta Λ_1 and Λ_2 are related semiclassically to the interaction radius of the colliding nuclei while the corresponding widths Δ_1 and Δ_2 are related to the thickness of the region in which the nuclear interaction between the colliding nuclei takes place without destruction of the identity of either of the nuclei. The reduced radius $r_{1/2}$ and diffusivity d can be obtained by means of the semiclassical relations (Mermaz, 1985)

$$\Lambda_2=kR_{1/2}\left(1-\frac{2\eta}{kR_{1/2}}\right)^{1/2}$$

$$\Delta_2=kd\left(1-\frac{\eta}{kR_{1/2}}\right)\left(1-\frac{2\eta}{kR_{1/2}}\right)^{-1/2}, \qquad (4)$$

where $R_{1/2}=r_{1/2}(A_1^{1/3}+A_2^{1/3})$, k is the wave number and $\eta=mZ_1Z_2e^2/(\hbar^2k)$ the Sommerfeld parameter. The remaining parameter, μ, is required to introduce the strength of the nuclear phase shift.

The semiclassical approximation assumes that the contributions to the cross section come mainly from the large angular momenta. We consider the Legendre polynominals, taken as a special case of the associated Legendre functions. The asymptotic form of the Legendre functions can be written as (Amado et al., 1985)

$$P_l^m(\cos\theta)\approx\left(l+\frac{1}{2}\right)^m\left(\frac{\theta}{\sin\theta}\right)^{1/2}J_{-m}\left[\left(l+\frac{1}{2}\right)\theta\right] \qquad (5)$$

which is valid for all m and all angles except when $(\pi-\theta)\leq l^{-1}$. The scattering amplitude can now be written as an integral over the continuous variable $\lambda=l+\frac{1}{2}$, using the asymptotic form Eq.

(5) with m=0 and replacing S_l^N by a continous differential function $S_N(\lambda)$, as

$$f_N(\theta) \approx \frac{1}{ik} \left(\frac{\theta}{\sin\theta}\right)^{1/2} \int_{1/2}^{\infty} \lambda \exp[2i\sigma(\lambda)] [S_N(\lambda)-1] J_0(\lambda\theta)\, d\lambda \qquad (6)$$

This formula is similar to Eq. (5) of Amado et al.. We use a continuous variable λ and the McIntyre parametrization for $S_N(\lambda)$ instead of an impact parameter b and an eikonal form for S(b), repectively.

Results and Conclusions

The elastic scattering cross sections are calculated from the scattering amplitude Eqs. (1) and (6). The two numerical results of elastic scattering angular distributions using the McIntyre parametrization of S-matrix for ^{12}C+^{40}Ca and ^{12}C+^{90}Zr at $E_{lab}=300$ MeV are presented in Fig. 1. The solid and broken curves represent the calculated elastic differential scattering cross sections obtained by using Eqs. (1) and (6), respectively. In both cases, the two calculated results are agreed well with the observed data (Sahm et al., 1986). The parameters used in the calculations are listed in table 1. r_{ph} and d_{ph} in table 1 are the radius and diffusivity for the phase shift of S-matrix element corresponding to $r_{1/2}$ and d for the modulus of S-matrix element, respectively. The term, θ_g, is the grazing angle equal to $2\tan^{-1}(\eta/\Lambda_2)$. As expected, these angles and the total reaction cross sections σ_R increase as the target mass increases. It can be noticed in table 1 that the two grazing angular momenta, Λ's, and the corresponding widths, Δ's, increase as the target mass increases.

The moduli of the S-matrix elements for ^{12}C+^{40}Ca and ^{12}C+^{90}Zr systems at $E_{lab}=300$ MeV are plotted in Fig. 2 along with the deflection

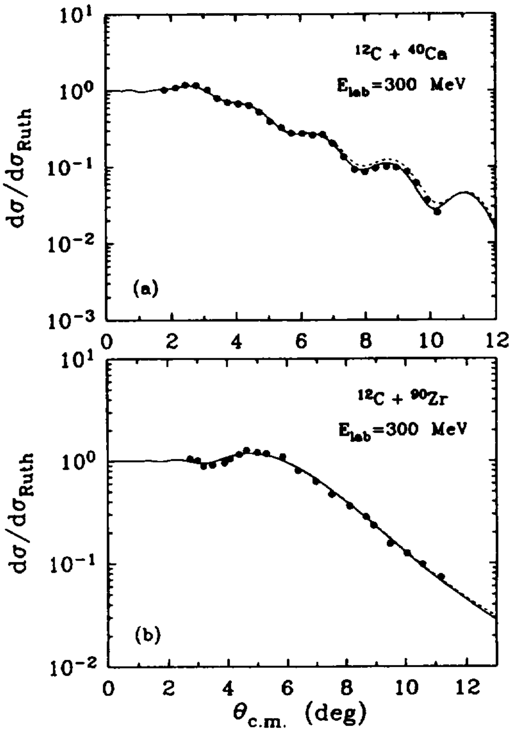

Fig. 1. Elastic scattering angular distributions for the scatterings (a) ^{12}C+^{40}Ca and (b) ^{12}C+^{90}Zr at $E_{lab}=300$ MeV. The solid circles denote the observed data (Sahm et al., 1986). Solid and broken curves are the calculated cross sections from Eqs. (1) and (6), respectively.

Table 1. McIntyre's phase shift analysis for ^{12}C ion elastic scattering at $E_{lab}=300$ MeV.

Target	^{40}Ca	^{90}Zr
$r_{1/2}$(fm)	1.371	1.261
d(fm)	0.491	0.779
μ	0.715	0.391
r_{ph}(fm)	1.325	1.471
d_{ph}(fm)	0.495	0.878
η	3.780	7.559
Λ_1	72.493	107.512
Δ_1	5.004	10.192
Λ_2	75.148	90.998
Δ_2	4.963	9.052
θ_g(°)	5.759	9.497
σ_R(mb)	2,020	2,413

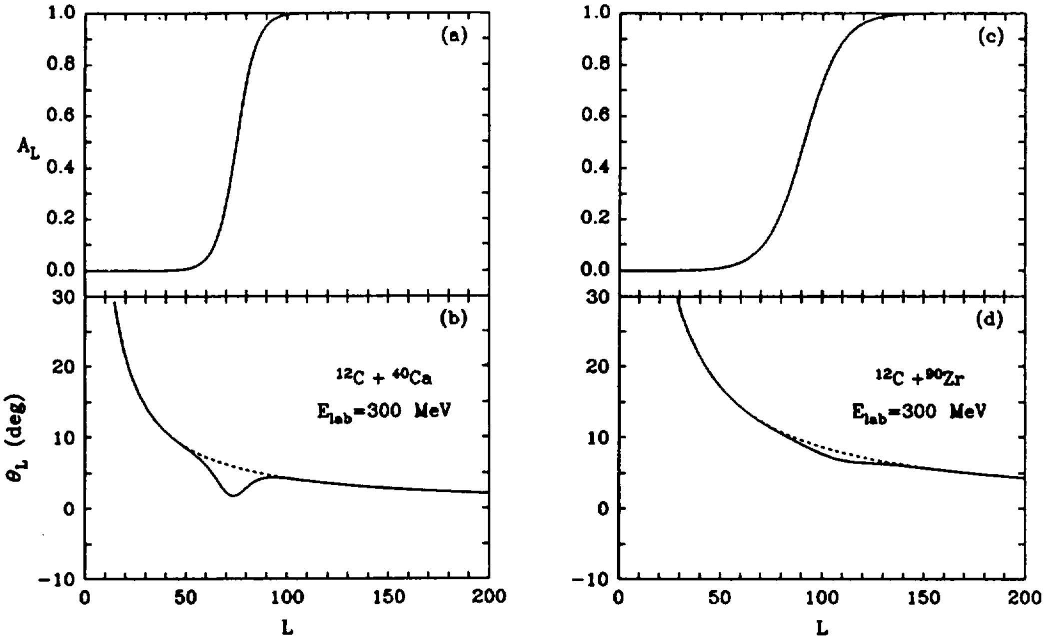

Fig. 2. Moduli of the S-matrix A_l and deflection functions θ_l for the systems $^{12}C+^{40}Ca$ and $^{12}C+^{90}Zr$ at $E_{lab}=300$ MeV plotted versus the orbital angular momentum. In (b) and (d), the broken curves represent the deflection functions for the Coulomb phase shift.

functions which are equal to twice the derivative of the Coulomb plus nuclear phase shift of formula (2). In this figure, we can see that the modulus of the S-matrix and deflection function are shifted to the right as the target mass increases. It can be also noticed that the deflection function for $^{12}C+^{40}Ca$ system displays a positive nuclear minimum angle ($\theta_{N.R.}=1.876°$), but the deflection function for $^{12}C+^{90}Zr$ system shows a similar structure to the case of pure Coulomb scattering.

Fig. 3 present the partial wave contributions, $\sigma_l = \frac{\pi}{k^2}(2l+1)(1-|S_l^N|^2)$, to the total reaction cross sections of $^{12}C+^{40}Ca$ and $^{12}C+^{90}Zr$ systems at $E_{lab}=300$ MeV as a function of orbital angular momentum. This figure shows that regions of higher partial waves almost never contribute to the total reaction cross section.

In conclusion, our calculated results using the McIntyre parametrization of S-matrix are

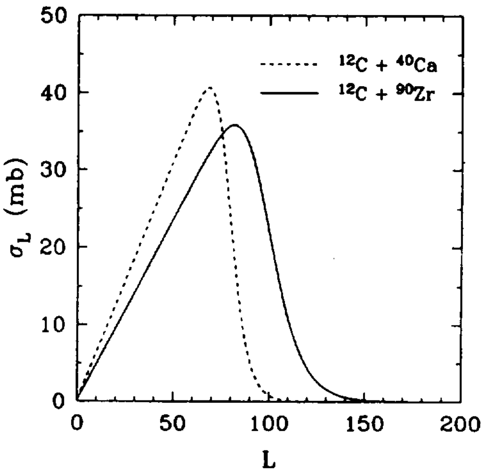

Fig. 3. Partial wave reaction cross sections, $\sigma_l = \frac{\pi}{k^2}(2l+1)(1-|S_l^N|^2)$, for the $^{12}C+^{40}Ca$ and $^{12}C+^{90}Zr$ systems at $E_{lab}=300$ MeV as a function of orbital angular momentum.

successful in reproducing the elastic scattering data of ^{12}C ions at $E_{lab}=300$ MeV from ^{40}Ca and ^{90}Zr targets. It is shown that the integral form for the scattering amplitude gives cross sections for the elastic scatterings of ^{12}C on ^{40}Ca and ^{90}Zr at $E_{lab}=300$ MeV which are in satisfactory agreement with those obtained from the direct sum of partial waves. It is also found that the deflection function for ^{12}C+^{40}Ca displays a positive nuclear minimum angle; meanwhile, the deflection function for ^{12}C+^{90}Zr shows a similar structure to the case of pure Coulomb scattering.

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〈국문초록〉

$E_{lab}=300$ MeV에서 ^{12}C 이온의 탄성산란에 대한 위상이동량 분석

산란행렬에 대한 McIntyre 파라미터에 기초한 위상이동량 분석을 나타내었다. 이 방법은 입사에너지가 300 MeV인 ^{12}C+^{40}Ca과 ^{12}C+^{90}Zr계의 탄성산란에 성공적으로 적용시킬 수 있었다.