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Theory of Magneto-Optical Absorption Lineshape Function for a Low Impurity Density Case

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저밀도 불순물에 대한 자기 광흡수 선모양함수 이론

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Summary

On the basis of the Kubo formalism for linear response, a theory of magneto-optical transitions in the electron-impurity system is presented for the Faraday configuration. The frequency-dependent conductivity of the system is evaluated by using the Mori-type projection technique. In the parabolic band model, the general lineshape functions which are applicable to both a weak and an arbitrary and/or strong electron-impurity coupling are introduced in two different ways. Explicit expressions for direct interband and intraband transition are given as functions of temperature, magnetic field, impurity concentration, and the incident photon frequency. The results are compared with those of some other authors.

1. Introduction

Magneto-optical transitions have been extensively studied as a powerful tool for investigating transport behavior of electrons in semiconductors. Especially, the shape of the line, the linewidth and shift of absorption peaks, and their dependence on temperature, magnetic field strength and impurity concentration were the object of study since their properties are very sensitive to the type of scattering mechanisms affecting the behavior of carriers as well as the band structure of solids. The absorption lineshapes in semiconductors are typically broadened by the scattering mechanisms including electron-electron and electron-background (impurity and phonon) interac-

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tions. But if the number density of electrons is very low as in semiconductors, the electron-electron interaction may be neglected. Then, among the kinds of interactions available, the background scatterings may be dealt with as perturbation.

In the presence of a constant magnetic field, the noninteracting electrons perform undisturbed cyclotron motion. The electron energy is made up of the kinetic energy of the original motion in the field direction, together with the quantized energy of the oscillatory motion in the plane perpendicular to the field direction. This results in the Landau splittings in both conduction and valence bands in solids.

In the process of absorption of photons the electrons change their energies and momenta to make optical transitions to higher sublevels. Usually we deal with two kinds of transitions. The first is intraband transitions including direct and indirect cyclotron transitions. The second is interband transitions including direct and indirect transitions(Madelung,1978).

Many theoretical(Argyres and Sigel,1974, Ciobanu and Banyai,1983, Choi and Chung,1983, 1984, Kawabata 1967, Lodder and Fujita,1968, Prasad 1982, Ryu and choi,1984, 1985, Royen et al. 1984, Shin et al. 1973, Suzuki and Dunn,1982, 1988, and Yi et al. 1987) as well as experimental studies(McCombe et al. 1976, Matsuda and Otsuka,1979, and Kobori et al. 1990) on these topics have appeared. But most of the work on the lineshape studies has been focused on intraband transitions, especially on cyclotron resonance lineshape and the effect of scattering associated with interband transitions has been paid less attention(Lynch,1985). It should be noted that the agreement between the experimental and theoretical results is not satisfactory(Royen et al. 1984 and Kobori et al. 1990) and that many theoretical investigations(Royen et al. 1984) based on the Kubo formalism(Kubo,1957) have produced a bewildering variety of results. Thus the situation on the quantum limit cyclotron transition still remains unclear.

The basic problem of discrepancy among these theories based on the Kubo for malism may be traced back to the ways of expanding the perturbative terms of the electron-background interactions. Considering many different methods(Prasad 1982 and Suzuki and Dunn,1982, 1988) presented so far, we see that the lineshape functions of the magneto-optical transitions are classified into two categories in general. The first of these is the case of a weak electron-background coupling, which is given in the closed form representation. The second is the case of an arbitrary and/or strong one, which is given in the iterative manner.

In the limit of weak electron-phonon coupling the present authors(Ryu and Choi,1984, 1985 and Yi et al. 1987) obtained the lineshape function given in the closed form representation, and showed that Kawabata's method (Kawabata 1967) and Argyres and Sigel's technique(Argyres and Sigel,1974) gave the same results in the theory of both the intraband(Ryu and Choi,1984) and the interband(Yi et al. 1987) magneto-optical transitions. The authors also suggested that there exist two weak coupling schemes in terms of the calculation with respect to the electron states and the phonon averaging.

The purpose of the present work is to present general quantum theories of magneto-optical absorption lineshape which are applicable to both a weak electron-impurity interaction and an arbitrary and/or strong one. The origin of this formalism dates back to the discovery of the theory of nonlinear static conductivity by the present authors(Ryu and Choi,1991). Here we apply the theory to the case in which the single-electron states are expressed in terms

of Bloch states. In the formulation, we shall adopt a parabolic band model. Both direct interband transition for direct gap materials and intraband transition shall be considered as an example. We assume that the effective mass of the carriers is isotropic in the vicinity of the band extrema $\vec{k} \to 0$, and neglect the band degeneracy as well as the effects due to spin and spin-orbit coupling. Based on the model, we will derive the general formulas for lineshape associated with magneto-optical transitions by using the Mori-type projection operator technique and apply them to the spectral region in which the two types of transitions are observed.

The present paper is organized as follows: In Sec.2, we will describe the model of the system. The frequency-dependent conductivity in the parabolic band model is evaluated in Sec.3. The conductivity is closely related to the lineshape function due to the collision process. In Sec.4, the general lineshape function is obtained in the band model by utilizing the Mori-type projection operator method presented by present authors(Ryu and Choi, 1991). In Sec.5, we will calculate the functions for both transitions. The general expressions of the lineshift and linewidth for a weak coupling and an arbitrary and/or strong one are given in both transition cases as functions of the temperature, magnetic field, impurity concentration, and the incident photon frequency. Comparison with some other theories shall be made in the last section.

2. Description of the system

Consider a system of N_e electrons in interaction with the impuritys, initially in equilibrium with a temperature T. Then, in the presence of a static magnetic field $\vec{B}$, the time-independent Hamiltonian H of the system can be written(Suzuki et al. 1988) as

$$H = H_e + H_{eB}. \tag{2.1}$$

$$H_e + H_{eB} = \sum_{s,s'} \sum_{\lambda,\lambda'} < \alpha_s | (h_e + h_{e\cdots}) | \lambda_{s'} > a^+_{\alpha_s} a_{\lambda_{s'}}, \tag{2.2}$$

$$h_e = [\vec{p} + e\vec{A}(\vec{r})]^2/2m^*_e, \tag{2.3}$$

$$h_{e\cdots} = \sum_{\vec{q}} \hat{v}(\vec{q}) exp(i\vec{q}\cdot\vec{r}), \tag{2.4}$$

where $| \alpha_s >$ means the electron state in the s-band: α denotes the Landau state $(N, \vec{k})$. $N(= 0, 1, 2, \cdots)$ and s are Landau level index and the band index, respectively. $a^+_{\alpha_s}(a_{\alpha_s})$ is the creation (annihilation) operator for an electron with momentum $\vec{p}$, effective mass m^*_s and energy E^s_α in Landau state α within the s-band. $\vec{r}$ is the position vector of the electron. $\vec{A}(\vec{r})$ is the vector potential which gives rise to the static magnetic field $B(= \nabla \times \vec{A})$. $\hat{v}(\vec{q})$ means the Fourier transform of the impurity potential. It is well-known that when a static magnetic field is applied along the z-direction of an isotropic semiconductor, characterized by the vector potential $\vec{A}(\vec{r}) = (0, Bx, 0)$, the Hamiltonian of the unperturbed single-electron is given by

$$h_e = [p^2_x + (p_y + m^*_e\omega_e x)^2 + p^2_z]/2m^*_e, \tag{2.5}$$

where $\omega_e = eB/m^*_e$ is the cyclotron frequency in the s-band. The eigenvalues E^s_α of the unperturbed single-electron Hamiltonian h_e and the single-electron state $| \alpha_s >$ in the s-band are, respectively, given by

$$E^s_\alpha = (N + 1/2)\hbar\omega_e + \epsilon_s(k_z), \tag{2.6}$$

$$\epsilon_s(k_z) \equiv \hbar^2 k^2_z/2m^*_e, \tag{2.7}$$

$$| \alpha_s > \equiv | N, \vec{k}, s > \equiv | N, k_y, k_z, s > = | U^s_0(\vec{r}) F^s_\alpha(\vec{r}) >, \tag{2.8}$$

where $U^s_0(\vec{r})$ is the Bloch function for the s-band at $\vec{k} = 0$ and the envelope function $F^s_\alpha(\vec{r})$ is given by

$$F_\alpha^s(\vec{r}) = (L_y L_z)^{-1/2}\phi_N(x + \hbar k_y/eB)\exp(i\vec{k}\cdot\vec{r}), \quad (2.9)$$

$$\phi_N(x) = (2^N N!\sqrt{\pi}r_0)^{-1/2}H_N(x/r_0)\exp(-x^2/2r_0^2). \quad (2.10)$$

Here L_y and L_z are the y- and z-directional normalization lengths, H_N is the Nth Hermite polynomial, and $r_0 \equiv (\hbar/eB)^{1/2}$. For band s' the function $F_\alpha^{s'}(\vec{r})$ will have the same form given by Eqs.(2.9) and (2.10) where $\vec{k}$ and N are replaced by $\vec{k}'$ and N', respectively. It should be noted that the cell periodic part of the Bloch function is assumed to be independent of electron wavevector $\vec{k}$ and the magnetic field. The envelope function $F_\alpha^s(\vec{r})$ is slowly varying while the Bloch function $U_0^s(\vec{r})$ is rapidly varying. Therefore it is noted that the $U_0^s(\vec{r})$'s are normalized in the unit cell whereas the $F_\alpha^s(\vec{r})$'s over the whole crystal:

$$\int_C U_0^{s'*}(\vec{r})U_0^s(\vec{r})d^3r = \delta_{ss'}, \quad (2.11a)$$

$$\int_\Omega F_\alpha^{s'*}(\vec{r})F_\lambda(\vec{r})d^3r \doteq \delta_{s\alpha} = \delta_{N,N'}\delta(k_y - k_{y'})\delta(k_z - k_{z'}), \quad (2.11b)$$

where C is the volume of the unit cell and $\Omega(= L_x L_y L_z)$ is that of the crystal in real space.

In the parabolic band model, the single-electron energy in the Landau state α in the conduction and the valence bands, respectively, are given by

$$E_\alpha^c = E_N^c(k_z) = (N + 1/2)\hbar\omega_c + \epsilon_c(k_z) + E_g, \quad (2.12a)$$

$$E_\alpha^v = E_{N'}^v(k_z') = -(N' + 1/2)\hbar\omega_v - \epsilon_v(k_z'). \quad (2.12b)$$

where $\omega_c \equiv eB/m_c^*$ and $\omega_v \equiv eB/m_v^*$. The subscripts c and v indicate the conduction and valence bands, respectively. We see from Eqs. (2.12a) and (2.12b) that in the presence of the magnetic field, the conduction and valence bands separated at $\vec{k} = 0$ by the direct-band-gap E_g are further split into Landau subbands specified by the Landau level indices $N(N') = 0, 1, 2, \cdots$, in which the energies of the single-electron in the conduction and the valence bands are quantized in the x-y plane and quasi-continuous in the z-direction. The minimum of the lowest subband of the conduction band occurs at the energy $\frac{1}{2}\hbar\omega_c$ above the energy band minimum in zero field, and the maximum of the highest subband in the valence band occurs at the energy $\frac{1}{2}\hbar\omega_v$ below the valence band maximum in zero field, thus increasing the band gap by $\frac{1}{2}\hbar(\omega_c + \omega_v)$. The changes in the energy levels due to the application of magnetic field and the transport properties of electrons in a solid can therefore be studied through the resulting alteration of its optical properties.

3. Theory of magneto-optical transition

When a circularly polarized electromagnetic wave of amplitude E and frequency ω given by

$$E_x = E\cos\omega t, \quad E_y = E\sin\omega t, \quad E_z = 0 \quad (3.1)$$

is applied along the z-axis in the semiconductor, the absorption power delivered to the system is given for the Faraday configuration ($E \perp B$) as(Kawabata 1967)

$$P = (E^2/2)Re[\sigma_+(\bar{\omega})], \quad (3.2)$$

where Re means "the real part of", $\bar{\omega} \equiv \omega - i\delta(\delta \to 0^+)$ and $\sigma_+(\bar{\omega})$ is the complex optical conductivity. Each elements of $\sigma_{kl}(\bar{\omega})$ $(k, l = x, y, z)$ is given in the Kubo formalism and $\sigma_{+-}(\bar{\omega})$ can be expressed as(Ryu et al. 1991)

$$\sigma_+(\bar{\omega}) = \Omega^{-1}\int_0^\infty dt\, \exp(-i\bar{\omega}t)\int_0^\beta d\beta_1 < Tr[\rho_c J^-(-i\hbar\beta_1 \mid H_+)J^+(t \mid H)] >_{imp} \quad (3.3a)$$

$$= \lim_{\omega_- \to 0}\frac{\partial}{\partial\omega_-}\Omega^{-1}\int_0^\infty dt\exp(-i\bar{\omega}t) < Tr[\rho_c(\vec{H})J^+(t \mid H)] >_{imp}. \quad (3.3b)$$

where $\beta = (k_B T)^{-1}$ for the temperature T, $< \cdots >_{imp}$ denotes the average over the impurity distribution, Tr means the many-body trace, $J^{\pm} \equiv J_x \pm iJ_y$ are the transverse components of the total current operator in the many-body formalism, $\tilde{J}(t \mid H)$ is the time-dependent total current operator in the Heisenberg representation, and

$$\rho_e = \frac{exp[\beta(\zeta N_e - H_e)]}{Tr^e\{exp[\beta(\zeta N_e - H_e)]\}}. \tag{3.4}$$

Here ζ is the chemical potential, $N_e = \sum_{\alpha_e} a^{+}_{\alpha_e} a_{\alpha_e}$ and Tr^e denotes the many-electron trace

To obtain Eq.(3.3b) we have used the following identity (Fujita 1968)

$$\int_0^{\beta} d\beta_1 \rho_e \tilde{J}(-i\hbar\beta_1 \mid H_e) = \lim_{\omega_- \to 0} \frac{\partial}{\partial \omega_-} \rho_e(\tilde{H}). \tag{3.5}$$

where $\tilde{H} \equiv H_e - \vec{u} \cdot \vec{J}$, $\vec{u}$ being a c-number vector, is the modified Hamiltonian. It should be noted that the total current operator can be written in terms of the single-electron current operator j as

$$J = \sum_{\alpha_e} \sum_{\lambda_{e'}} < \alpha_e \mid j \mid \lambda_{e'} > a^{+}_{\alpha_e} a_{\lambda_{e'}}. \tag{3.6}$$

Here j is the velocity operator multiplied by the electronic charge $-e$.

Then, the frequency-dependent conductivity formula, Eq.(3.3b), can be expressed in terms of the single-electron trace as (Ryu et al. 1984)

$$\sigma_{+-}(\omega) = \Omega^{-1} \int_0^{\infty} dt \; exp(-i\omega t) < tr[\lim_{\omega_- \to 0} \frac{\partial f}{\partial \omega_-} j^{+}(t \mid h_e + h_{e-i})] >_{imp}, \tag{3.7}$$

where h_{e-i} is the scattering potential in the single-electron representation, and f is the modified Fermi-Dirac operator given by

$$f \equiv [exp\{\beta(h_e + \vec{u} \cdot \vec{j} - \zeta)\} + 1]^{-1}. \tag{3.8}$$

In order to rewrite Eq.(3.7) in more convenient form, we rewrite the interaction term in the background average as

$$\begin{aligned} & tr[\lim_{\omega_- \to 0} (\frac{\partial f}{\partial \omega_-}) j^{+}(t \mid h_e + h_{e-i})] \\ &= \sum_{\alpha_e \lambda_{e'}} \frac{1}{2\pi i} \oint_C dz f(z) < \alpha_e \mid R_z j^{-} R_z \mid \lambda_{e'} > \\ & \times < \lambda_{e'} \mid j^{+}(t \mid h_e + h_{e-i}) \mid \alpha_e >, \end{aligned} \tag{3.9}$$

where we have used

$$\lim_{\omega_- \to 0} \frac{\partial}{\partial \omega_-} (z - h_e - \vec{u} \cdot \vec{j})^{-1} = R_z j \; R_z \tag{3.10}$$

with $R_z = (z - h_e)^{-1}$ and $f(z)$ is defined by

$$f(z) = [exp\{\beta(z - \zeta)\} + 1]^{-1}. \tag{3.11}$$

Then, by considering Eqs.(3.7) and (3.9) the frequency-dependent conducti vity is reduced to

$$\sigma_{+-}(\omega) = \Omega^{-1} \sum_{\alpha_e \lambda_{e'}} << \alpha_e \mid Y_e \mid \lambda_{e'} >< \lambda_{e'} \mid \tilde{j}^{+}(\omega) \mid \alpha_e >>_{imp}, \tag{3.12}$$

where $\tilde{j}^{+}(\omega)$ is the Fourier-Laplace transform (FLT) of $j^{+}(t \mid h_T)$ defined by

$$\tilde{j}^{+}(\omega) \equiv FLT[j^{+}(t \mid h_T)] = \int_0^{\infty} dt \; exp(-i\omega t) j^{+}(t \mid h_T) \tag{3.13}$$

with $h_T \equiv h_e + h_{e-i}$ and

$$< \alpha_e \mid Y_e \mid \lambda_{e'} > = \frac{f(E^e_{\alpha}) - f(E^{e'}_{\lambda})}{E^{e'}_{\lambda} - E^e_{\alpha}} < \alpha_e \mid j^{-} \mid \lambda_{e'} >. \tag{3.14}$$

where $f(E^e_{\alpha})$ denotes the Fermi-Dirac dist ribution for single-electrons in the s-band and the electron energy E^e_{α}. Then, the frequency-dependent conductivity formula can be rewritten from Eqs.(3.12) and (3.14) as

$$\begin{aligned} \sigma_{+-}(\omega) &= \Omega^{-1} \sum_{\alpha_e \lambda_{e'}} \frac{f(E^e_{\alpha}) - f(E^{e'}_{\lambda})}{E^{e'}_{\lambda} - E^e_{\alpha}} < \alpha_e \mid j^{-} \mid \lambda_{e'} > \\ & \times << \lambda_{e'} \mid \tilde{j}^{+}(\omega) \mid \alpha_e >>_{imp}, \end{aligned} \tag{3.15}$$

where $< \alpha_e \mid j^{-} \mid \lambda_{e'} > = [< \lambda_{e'} \mid j^{+} \mid \alpha_e >]^{*}$. For intraband cyclotron transition and interband transition, the selection rules are given by (Burstein et al.,1959)

$$\begin{aligned} & (< \lambda_{e'} \mid j^{+} \mid \alpha_e >)_{intra} \\ &= \int_C U^{e'*}_{\delta}(\vec{r}) U^{e}_{\delta}(\vec{r}) d^3r \int_{\Omega} F^{*}_{\lambda}(\vec{r}) j^{+} F_{\alpha}(\vec{r}) d^3r \\ &= \int_{\Omega} F^{*}_{\lambda}(\vec{r}) j^{+} F_{\alpha}(\vec{r}) d^3r \delta_{\alpha,\alpha'} \delta_{\lambda,\alpha+1} \\ &\equiv < \alpha + 1 \mid j^{+} \mid \alpha > \delta_{\alpha,\alpha'} \equiv j^{+}_{\alpha+1\alpha} \delta_{\alpha,\alpha'}. \end{aligned} \tag{3.16}$$

$$\begin{aligned} & (< \lambda_{e'} \mid j^{+} \mid \alpha_e >)_{inter} \\ &= \int_C U^{e'*}_{\delta}(\vec{r}) j^{+} U^{e}_{\delta}(\vec{r}) d^3r \int_{\Omega} F^{*}_{\lambda}(\vec{r}) F_{\alpha}(\vec{r}) d^3r \\ &= \int_C U^{e'*}_{\delta}(\vec{r}) j^{+} U^{e}_{\delta}(\vec{r}) d^3r \delta_{\alpha,\lambda} \\ &\equiv < U^{e'}_{\delta}(\vec{r}) \mid j^{+} \mid U^{e}_{\delta}(\vec{r}) > \delta_{\alpha,\lambda} \equiv j^{+}_{\lambda',\alpha} \delta_{\alpha,\lambda}. \end{aligned} \tag{3.17}$$

To evaluate $< \tilde{j}^{+}_{\alpha+1\alpha}(\bar{\omega}) >_{imp}$ of Eq.(3.15) for the intraband transition, many techniques were presented, such as the double time Green's-function approach (Ciobanu et al. 1963), Kawabata's projection operator method (Kawabata,1967), the proper connected daigram method (Lo d-der et al.1968), Nakajima projection operator method (Shin et al. 1973), Argyres-Sigel's projection operator method (Argyres,1974), the coherent potential approximation method (Prasad 1982), and the resolvent-superoperator approach (Suzuki et al. 1982, 1988) Note that many methods (Choi et al. 1984, Yi et al. 1987, and Suzuki 1988), were introduced for the interband transition. The central problem of evaluation of Eq.(3.15) is the evaluation of the configuration over the impurity fields. The main task is to give a suitable expansion method for the operators $<< \lambda_{\sigma'} \mid \tilde{j}^{+}(\bar{\omega}) \mid \alpha_{\sigma} >>_{imp}$ in Eq.(3.15), which will be outlined in the next section.

4. Lineshape function

In order to obtain the lineshape function we will present two representations using the Mori-type projection operator technique (Ryu et al. 1991) a closed-form representation and a continued-fraction form representation.

A. The closed form

For the calculation of $<< \lambda_{\sigma'} \mid \tilde{j}^{+}(\bar{\omega}) \mid \alpha_{\sigma} >>_{imp}$ in Eq(3.15), we define the projection operators P_0 and P_0' for both the single-electron states $\mid \alpha_{\sigma} >$ and $\mid \lambda_{\sigma'} >$ and the impurity averaging as

$$P_0 X = [X_{fi}/j^{+}_{fi}]j^{+}, \tag{4.1}$$

$$P_0' = 1 - P_0, \tag{4.2}$$

where $X_{fi} \equiv << \lambda_{\sigma'} \mid X \mid \alpha_{\sigma} >>_{imp}$ for any operator X.

Following Mori (Mori 1965), we separate $j^{+}(t \mid h_T)$ into the projective and vertical components with respect to the j^{+}-axis as

$$\begin{aligned} & j^{+}(t \mid h_T) \\ &= P_0 j^{+}(t \mid h_T) + P_0' j^{+}(t \mid h_T) \\ &= Z_{0fi}(t \mid h_T) j^{+} + \int_0^t dt_1 Z_{0fi}(t_1 \mid h_T) f_i'(t - t_1 \mid h_T), \end{aligned} \tag{4.3}$$

where

$$Z_{0fi}(t \mid h_T) \equiv j^{+}_{fi}(t \mid h_T)/j^{+}_{fi}, \tag{4.4}$$

$$f_i'(t \mid h_T) \equiv \exp(iL_1 t/\hbar) f_i', \tag{4.5}$$

$$f_i' \equiv iL_1 j^{+}/\hbar, \tag{4.6}$$

$$L_1 \equiv P_0' L_T, \tag{4.7}$$

$$L_T \equiv L_e + L_{eB}. \tag{4.8}$$

Here L_e and L_B are the Liouville operators corresponding to the single-electron Hamiltonian h_e, and the scattering potential h_{e-i}, respectively.

In order to obtain $\tilde{j}^{+}_{fi}(\bar{\omega})$ or $\tilde{Z}_{0fi}(\bar{\omega})$, we differentiate Eq.(4.4) as

$$\begin{aligned} & \frac{d}{dt} Z_{0fi}(t \mid h_T) \\ &= i\omega_{0fi} Z_{0fi}(t \mid h_T) + \int_0^t dt_1 \Delta_{0fi}(t - t_1 \mid h_T) Z_{0fi}(t_1 \mid h_T), \end{aligned} \tag{4.9a}$$

$$= i\omega_{0fi} Z_{0fi}(t \mid h_T) + \int_0^t dt_1 Z_{1fi}(t - t_1 \mid h_T) \Delta_{0fi} Z_{0fi}(t_1 \mid h_T). \tag{4.9b}$$

Here

$$\omega_{0fi} \equiv (L_T j^{+}/\hbar)_{fi}/j^{+}_{fi} = (E_f - E_i)/\hbar, \tag{4.10}$$

$$\Delta_{0fi}(t \mid h_T) \equiv f_{1fi}(t \mid h_T)/j^{+}_{fi} \equiv Z_{1fi}(t \mid h_T)\Delta_{0fi}, \tag{4.11}$$

$$f_i(t \mid h_T) = iL_T f_i'(t \mid h_T)/\hbar, \tag{4.12}$$

$$Z_{1fi}(t \mid h_T) \equiv f_{1fi}(t \mid h_T)/f_{1fi}, \tag{4.13}$$

$$\Delta_{0fi} \equiv f_{1fi}/j^{+}_{fi}, \tag{4.14}$$

where we have used $< (h_{eB})_{ff} - (h_{eB})_{ii} >_{imp} = 0$ in Eq.(4.10).

Then, the FLT of Eqs.(4.9a) and (4.9b) leads to

$$\tilde{Z}_{0fi}(\omega) \equiv \tilde{j}^{+}_{fi}(\omega)/j^{+}_{fi} = [i\omega - i\omega_{0fi} + \tilde{\Sigma}_{0fi}(\omega)]^{-1}. \tag{4.15}$$

Here $\tilde{\Sigma}_{0fi}(\bar{\omega})$, often called the lineshape function due to the electron-impurity interaction, is defined as

$$\tilde{\Sigma}_{0f_i}(\bar{\omega}) = -\tilde{\Delta}_{0f_i}(\bar{\omega}) \quad (4.16a)$$
$$= -\tilde{Z}_{1f_i}(\bar{\omega})\Delta_{0f_i}. \quad (4.16b)$$

where $\tilde{\Delta}_{0fi}(\bar{\omega})$ and $\tilde{Z}_{1fi}(\bar{\omega})$ are the Fourier-Laplace transform of Eqs.(4.11) and (4.13), respectively. Considering Eqs.(4.5)-(4.8), (4.11), (4.12) and (4.16a), and taking into account the relation $P_0L_eG_0P_0'X = (L_eG_0P_0'X)_{fi} = 0$ we obtain

$$\tilde{\Sigma}_{0f_i}(\bar{\omega}) = (i\hbar f_i')^{-1}\sum_{N=1}^{\infty}[(L_{e-}G_0P_0')^NL_TJ^+]_{fi}. \quad (4.17)$$

where $G_0 = (\hbar\bar{\omega} - L_e)^{-1}$ and we have used the relation $(A-B)^{-1} = A^{-1}\sum_{m=0}^{\infty}(BA^{-1})^m$ for any operators A and B. Now the lineshape function, $\tilde{\Sigma}_{0fi}(\bar{\omega})$, has been expanded with respect to L_B corresponding to the scattering potential. Eq.(4.17) is the general formula for lineshape given in a closed expansion form for electron-impurity systems, which is applicable to the weak coupling case since we have taken the relation $(A-B)^{-1} = A^{-1}\sum_{m=0}^{\infty}(BA^{-1})^m$ Eq.(4.17) is identical with the result (Argyres et al. 1974 and Choi et al. 1983) obtained by Argyres-Sigel's projection operator method. In Eq.(4.17), Argyres et al.(1974) and Choi et al.(1983) replaced L_T by L_B under the assumption that $P_0'L_eX = 0$ for any operator X. It should be noted that the condition is not always satisfied. In the case of the intraband transition the lineshape function obtained to the second order scattering strength gives same results whether the condition is used or not. The results obtained by Choi et al.(1984) and Suzuki(1988) for a weak coupling to the second order scattering strength in the interband transition are different from those of the present authors. The detailed description will be given in Sec. 5.

B. The continued-fraction form

In order to obtain $\tilde{Z}_{1fi}(\bar{\omega})$ of Eq.(4.16b) we define the projection operators P_1 and P_1' as

$$P_1X = (X_{fi}/f_{1fi})f_1, \quad (4.18)$$

$$P_1' = 1 - P_1. \quad (4.19)$$

By utilizing these operators we separate $f_1(t \mid h_T)$ of Eq.(4.12) into the projective and vertical components with respect to the f_1-axis as

$$\begin{aligned} &f_1(t \mid h_T) \\ &= P_1f_1(t \mid h_T) + P_1'f_1(t \mid h_T) \\ &= Z_{1fi}(t \mid h_T)f_1 + \int_0^t Z_{1fi}(t_1 \mid h_T)f_2'(t-t_1 \mid h_T)dt_1, \end{aligned} \quad (4.20)$$

where

$$f_2'(t \mid h_T) \equiv \exp(iL_2t/\hbar)f_2', \quad (4.21)$$

$$f_2' \equiv iL_2f_1/\hbar, \quad (4.22)$$

$$L_2 \equiv P_1'L_TP_0'. \quad (4.23)$$

In order to obtain $\tilde{Z}_{1fi}(\bar{\omega})$, we differentiate Eq.(4.13) as

$$\begin{aligned} &\frac{d}{dt}Z_{1fi}(t \mid h_T) \\ &= i\omega_{1fi}Z_{1fi}(t \mid h_T) + \int_0^t dt_1\Delta_{1fi}(t-t_1 \mid h_T)Z_{1fi}(t_1 \mid h_T) \quad (4.24a) \\ &= i\omega_{1fi}Z_{1fi}(t \mid h_T) + \int_0^t dt_1Z_{2fi}(t-t_1 \mid h_T)\Delta_{1fi}Z_{1fi}(t_1 \mid h_T). \quad (4.24b) \end{aligned}$$

where

$$\omega_{1fi} \equiv (L_TP_0'f_1/\hbar)_{fi}/f_{1fi}, \quad (4.25)$$

$$\Delta_{1fi}(t \mid h_T) \equiv f_{2fi}(t \mid h_T)/f_{1fi} \equiv Z_{2fi}(t \mid h_T)\Delta_{1fi}. \quad (4.26)$$

$$f_2(t \mid h_T) = iL_TP_0'f_2'(t \mid h_T)/\hbar, \quad (4.27)$$

$$Z_{2fi}(t \mid h_T) \equiv f_{2fi}(t)/f_{2fi}, \quad (4.28)$$

$$\Delta_{1fi} \equiv f_{2fi}/f_{1fi}. \quad (4.29)$$

Then the FLT of Eq.(4.24) leads to

$$\tilde{Z}_{1fi}(\bar{\omega}) \equiv \tilde{f}_{1fi}(\bar{\omega})/f_{1fi} = [i\bar{\omega} - i\omega_{1fi} + \tilde{\Sigma}_{1fi}(\bar{\omega})]^{-1}. \quad (4.30)$$

Here $\tilde{\Sigma}_{1fi}(\bar{\omega})$ is the first-order term in the continued-fraction forms given by

$$\tilde{\Sigma}_{1fi}(\bar{\omega}) = -\tilde{\Delta}_{1fi}(\bar{\omega}) \quad (4.31a)$$
$$= -\tilde{Z}_{2fi}(\bar{\omega})\Delta_{1fi}. \quad (4.31b)$$

We now see that $\tilde{\Delta}_{1fi}(\bar{\omega})$, the FLT of Eq.(4.26), is given in a closed-form expansion while $\tilde{Z}_{2fi}(\bar{\omega})$, the FLT of Eq.(4.28), can be given in a continued-

fraction manner via the successive projection operators onto the f_2, f_3, f_4 $\cdots$ axes as follows. In order to obtain the general form for $\tilde{Z}_{jfi}(\bar{\omega})$ we define the projection operators P_j and P_j' onto the f_j axis as

$$P_jX = (X_{fi}/f_{jfi})f_j, \tag{4.32}$$

$$P_j' = 1 - P_j. \tag{4.33}$$

Thus we have

$$\begin{aligned} & f_j(t \mid h_T) \\ &= iL_T\Pi_{m=0}^{j-2}P_m' f_j'(t \mid h_T)/\hbar \\ &= P_jf_j(t \mid h_T) + P_j'f_j(t \mid h_T) \\ &= Z_{jfi}(t \mid h_T)f_j + \int_0^t Z_{jfi}(t_1 \mid h_T)f_{j+1}'(t-t_1 \mid h_T)dt_1, \end{aligned} \tag{4.34}$$

where the notation $\Pi_{m=0}^{j-2}P_m'$ means $P_0'P_1'P_2'\cdots P_{j-2}'$ and

$$Z_{jfi}(t \mid h_T) = f_{jfi}(t \mid h_T)/f_{jfi}, \tag{4.35}$$

$$f_{j+1}'(t \mid h_T) \equiv exp(iL_{j+1}t/\hbar)f_{j+1}', \tag{4.36}$$

$$f_{j+1}' \equiv iL_{j+1}f_j/\hbar, \tag{4.37}$$

$$L_{j+1} \equiv P_j'L_T\Pi_{m=0}^{j-1}P_m'. \tag{4.38}$$

Then the time derivative of $Z_{jfi}(t \mid h_T)$ leads to

$$\begin{aligned} & \frac{d}{dt}Z_{jfi}(t \mid h_T) \\ &= i\omega_{jfi}Z_{jfi}(t \mid h_T) + \int_0^t dt_1\Delta_{jfi}(t-t_1 \mid h_T)Z_{jfi}(t_1 \mid h_T) \qquad (4.39a) \\ &= i\omega_{jfi}Z_{jfi}(t \mid h_T) + \int_0^t dt_1 Z_{j+1fi}(t-t_1 \mid h_T)\Delta_{jfi}Z_{jfi}(t_1 \mid h_T), \qquad (4.39b) \end{aligned}$$

where

$$\omega_{jfi} \equiv [L_T\Pi_{m=0}^{j-1}P_m' f_j/\hbar]_{fi}/f_{jfi}, \tag{4.40}$$

$$\Delta_{jfi}(t \mid h_T) \equiv f_{j+1fi}(t \mid h_T)/f_{jfi} \equiv Z_{j+1fi}(t \mid h_T)\Delta_{jfi}, \tag{4.41}$$

$$f_{j+1}(t \mid h_T) = iL_T\Pi_{m=0}^{j-1}P_m' f_{j+1}'(t \mid h_T)/\hbar, \tag{4.42}$$

$$\Delta_{jfi} \equiv f_{j+1fi}/f_{jfi}. \tag{4.43}$$

The FLT of Eq.(4.39) leads to

$$\begin{aligned} \tilde{Z}_{jfi}(\omega) &\equiv \tilde{f}_{jfi}(\bar{\omega})/f_{jfi} = [i\omega - i\omega_{jfi} - \tilde{\Delta}_{jfi}(\omega)]^{-1} \qquad (4.44a) \\ &= [i\omega - i\omega_{jfi} - Z_{j+1fi}(\omega)\Delta_{jfi}]^{-1}, \ (0 \le j \le \infty) \qquad (4.44b) \end{aligned}$$

where $\tilde{\Delta}_{jfi}(\bar{\omega})$ and $\tilde{Z}_{j+1fi}(\bar{\omega})$ are the FLT of $\tilde{\Delta}_{jfi}(t \mid h_T)$ and $\tilde{Z}_{j+1fi}(t \mid h_T)$, respectively. Now Eq.(4.44a) is given in a closed-form expansion in the j-th continued-fraction representation. By considering Eqs.(4.15), (4.16b), (4.30), (4.31b) and (4.44b), we obtain the general lineshape function given in a continued-fraction form:

$$\begin{aligned} & \tilde{\Sigma}_{0fi}(\bar{\omega}) \\ &= \cfrac{-\Delta_{0fi}}{i\omega - i\omega_{1fi} + \tilde{\Sigma}_{1fi}(\omega)} \\ &= \cfrac{-\Delta_{0fi}}{i\omega - i\omega_{1fi} - \cfrac{\Delta_{1fi}}{i\omega - i\omega_{2fi} - \cfrac{\Delta_{2fi}}{i\omega - i\omega_{3fi} - \cfrac{\Delta_{3fi}}{i\omega - i\omega_{4fi} - \cdots}}}} \end{aligned} \tag{4.45}$$

where Δ_{0fi}, Δ_{1fi}, $\cdots$ and ω_{1fi}, ω_{2fi}, $\cdots$ can be easily obtained from Eqs.(4.40) and (4.43). Note that $\tilde{\Sigma}_{1fi}(\bar{\omega})$ of Eq(4.45) is given in two forms expressed by the infinite closed-form expansion of the finite continued fraction order and the infinite continued fraction representation, and that any approximation is not taken to derive Eq(4.45). We see that Eq.(4.45) is applicable to the strong coupling case.

Considering Eqs.(3.15), (4.10) and (4.15) we can express the frequency-dependent magnetooptical conductivity tensor as

$$\sigma_{\alpha\beta}(\omega) = \frac{i\hbar}{\Omega}\sum_{fi}\frac{f(E_f) - f(E_i)}{E_f - E_i} \times \frac{|x_{fi}|^2}{\hbar\omega - E_f + E_i - < i\hbar\tilde{\Sigma}_{0fi}(\omega) >_B}. \tag{4.46}$$

It should be noted that the $\mid i >$ ($\equiv \mid \alpha_i >$) and the E_i ($\equiv E_\alpha^i$) are, respectively, the single-electron states and the eigenvalues of h_e. The lineshape function, $i\hbar\tilde{\Sigma}_{0fi}(\bar{\omega})$, results in the lifetime broadening, which is responsible for the spectral broadening of lineshape. Therefore, the real and imaginary parts of $i\hbar\tilde{\Sigma}_{0fi}(\bar{\omega})$ defined by

$$i\hbar\tilde{\Sigma}_{0fi}(\bar{\omega}) \equiv \hbar\tilde{\Psi}_{0fi}(\omega) + i\hbar\tilde{\Gamma}_{0fi}(\omega) \tag{4.47}$$

are, respectively, the lineshift and linewidth for the transition arising from the resonant absorption of a single photon of frequency ω between single-electron states $\mid i >$ and $\mid f >$. Real and imaginary parts of Eq(4.47) are of basic interest and are related to the quantities measured experimentally. The real part

provides the resonance shifting. The imaginary part gives directly the average value of the relaxation time, the inverse of which then measures the resonance broadening of the absorption spectrum. It should be noted that both of these quantities are functions of temperature, the magnetic field, impurity concentration, and the incident photon frequency.

For the one-band model, especially for the conduction band only, the selection rule is given by $j^{+}_{\lambda',\alpha'} \equiv j^{+}_{\alpha+1\alpha}\delta_{\alpha',\alpha}\delta_{\lambda,\alpha+1}$ in Eq.(3.16). Considering the transitions within this single band, Eq.(4.46) is given for the intraband magneto-optical absorption as

$$\sigma_{+-}(\omega) = \frac{i\hbar}{\Omega}\sum_{\alpha}\frac{f(E_{\alpha+1})-f(E_{\alpha})}{E_{\alpha+1}-E_{\alpha}} \times \frac{|j^{+}_{\alpha+1\alpha}|^2}{\hbar\omega - E_{\alpha+1}+E_{\alpha}-i\hbar\tilde{\Sigma}_{\alpha\alpha+1\alpha}(\omega)}, \quad (4.48)$$

where $E^{c}_{\alpha} \equiv E_{\alpha}$ is given by Eq.(2.6).

For the two-band model, the selection rule is given by $j^{+}_{\lambda'\alpha'} = j^{+}_{\lambda'\alpha'}\delta_{\lambda,\alpha}$ in Eq.(3.17). Therefore Eq.(4.46) is given for the direct interband magneto-optical absorption as

$$\sigma_{+-}(\omega) = \frac{i\hbar}{\Omega}\sum_{\alpha',\alpha}\frac{f(E^{v}_{\alpha})-f(E^{c}_{\alpha'})}{E^{c}_{\alpha'}-E^{v}_{\alpha}} \times \frac{|j^{+}_{\alpha'\alpha}|^2}{\hbar\omega - E^{c}_{\alpha'}+E^{v}_{\alpha}-i\hbar\tilde{\Sigma}_{\alpha\alpha'\alpha}(\omega)}, \quad (4.49)$$

where E^{c}_{α} and E^{v}_{α} are given by Eqs.(2.12a) and (2.12b), respectively. As seen from this, the onset of strong absorption due to the existence of the complex lineshape is shifted from the energy $\hbar\omega = \hbar^2k^2/2\mu + E_g + \hbar < \vec{k}_c \mid \tilde{\nabla}_0(\omega) \mid \vec{k}_v >$ in the absence of the magnetic field, to

$$\hbar\omega_{+-} = E^{c}_{\alpha'} - E^{v}_{\alpha} + \hbar\tilde{\nabla}_{0\alpha'\alpha}(\omega) = \hbar\omega_l + \hbar^2k_z^2/2\mu + E_g + \hbar\tilde{\nabla}_{0\alpha'\alpha}(\omega) \quad (4.50)$$

when the magnetic field is applied to the system. Here ω_l and μ are, respectively, given by

$$\omega_l \equiv (N+1/2)(\omega_c + \omega_v) = (N+1/2)eB/\mu. \quad (4.51)$$

$$\mu^{-1} \equiv m_c^{*-1} + m_v^{*-1}. \quad (4.52)$$

It is possible to determine the reduced mass μ, the band gap E_g, the energy shift $\hbar\tilde{\nabla}_{0\alpha'\alpha}(\omega)$ and the broadening $\hbar\tilde{\Gamma}_{0\alpha'\alpha}(\omega)$ in the absorption edges in the fundamental absorption experimentally by changing the magnetic field. Therefore the practical evaluation of the lineshape function is needed for both intraband and direct interband transitions, which will be given and compared with other theories in the next section.

5. Explicit Expression for the lineshape function

In this section we shall derive an explicit expression of the lineshape function for both weak coupling case and strong coupling one given in Eqs.(4.17) and (4.45), respectively. The central interest in the evaluation of Eqs.(4.17) and (4.45) is aver aging over the impurity configurations.

A. Weak coupling case

For the second order of the scattering potential in Eq.(4.17) we obtain the line shape function for the impurity scatterings as follows:

$$\begin{aligned}
&i\hbar\tilde{\Sigma}_{0\alpha'\alpha}(\omega) \\
&= < \sum_{\lambda(\neq\alpha)} \Big[\frac{(h_{\alpha'-1})_{\alpha'\lambda}\{(h_{\alpha'-1})_{\lambda\alpha} - (h_{\alpha-1})_{\lambda\alpha}j^{+}_{\lambda\lambda}/j^{+}_{\alpha\alpha}\}}{\hbar\omega - E^{c}_{\lambda} + E^{v}_{\alpha}} \\
&+ \frac{\{(h_{\alpha'-1})_{\alpha'\lambda} - (h_{\alpha'-1})_{\alpha'\lambda}j^{+}_{\lambda\lambda}/j^{+}_{\alpha'\alpha'}\}(h_{\alpha-1})_{\lambda\alpha}}{\hbar\omega - E^{c}_{\alpha'} + E^{v}_{\lambda}} \\
&+ \frac{(h_{\alpha'-1})_{\alpha'\lambda}\{(E^{c}_{\lambda} - E^{c}_{\alpha'}) - (E^{v}_{\lambda} - E^{v}_{\alpha})\}(h_{\alpha'-1})_{\lambda\alpha}j^{+}_{\lambda\lambda}/j^{+}_{\alpha'\alpha'}}{(\hbar\omega - E^{c}_{\lambda} + E^{v}_{\alpha})(\hbar\omega - E^{c}_{\lambda} + E^{v}_{\lambda})} \\
&+ \frac{(h_{\alpha'-1})_{\alpha'\lambda}\{(E^{c}_{\lambda} - E^{c}_{\alpha'}) - (E^{v}_{\lambda} - E^{v}_{\alpha})\}(h_{\alpha'-1})_{\lambda\alpha}j^{+}_{\lambda\lambda}/j^{+}_{\alpha'\alpha'}}{(\hbar\omega - E^{c}_{\alpha'} + E^{v}_{\lambda})(\hbar\omega - E^{c}_{\lambda} + E^{v}_{\lambda})} \Big] >_{imp}
\end{aligned}$$

(interband transition). (5.1)

$$\begin{aligned}
&i\hbar\tilde{\Sigma}_{0\alpha+1\alpha}(\omega) \\
&= < \sum_{\lambda(\neq\alpha+1)} \frac{(h_{\alpha'-1})_{\alpha+1\lambda}\{(h_{\alpha'-1})_{\lambda\alpha+1} - (h_{\alpha'-1})_{\lambda-\alpha+1\lambda-1}j^{+}_{\lambda-1\alpha}\}}{\hbar\omega - E_{\lambda} + E_{\alpha}} \\
&+ \sum_{\lambda(\neq\alpha)} \frac{\{(h_{\alpha'-1})_{\alpha\lambda} - (h_{\alpha'-1})_{\alpha+1\lambda+1}j^{+}_{\lambda+1\lambda}/j^{+}_{\lambda+1\alpha}\}(h_{\alpha'-1})_{\lambda\alpha}}{\hbar\omega - E_{\alpha+1} + E_{\lambda}} >_{imp}
\end{aligned}$$

(intraband transition). (5.2)

where we have used the relations

$$\sum_{\lambda_c}(P^{c}_{0}X)_{\lambda\alpha_c} = \sum_{\lambda(\neq\alpha)} X_{\lambda\alpha_c}, \quad (5.3)$$

$$\sum_{\lambda}(P^{v}_{0}X)_{\lambda\alpha} = \sum_{\lambda(\neq\alpha+1)} X_{\lambda\alpha}, \quad (5.4)$$

$$(G_0X)_{\lambda_\alpha\alpha} = \frac{X_{\lambda_\alpha\alpha}}{[\hbar\bar{\omega} - E^+_\lambda + E^+_\alpha]}, \tag{5.5}$$

$$(G_0X)_{\lambda\alpha} = \frac{X_{\lambda\alpha}}{[\hbar\bar{\omega} - E_\lambda + E_\alpha]}, \tag{5.6}$$

$$G_\alpha = [\hbar\bar{\omega} - L_\alpha]^{-1} \tag{5.7}$$

for any operator X. Eq.(5.2) is identical with those of Argyres et al.(1974) and Kawabata(1967) obtained for sufficiently weak scattering. In the case of $j^+_{\lambda\lambda-1}/j^+_{\alpha+1\alpha} \approx j^+_{\lambda+1\lambda}/j^+_{\alpha+1\alpha} \simeq 0$, Eq.(5.2) is reduced to those of Ciobanu et al.(1963) and Prasad et al.(1982) based on the double-time Green function approach. Note that these equations are good for sufficiently weak scattering which neglects the many-body coherence effect.

B. Strong coupling case

By considering Eq.(4.45) given in a continued-fraction manner we can obtain the general formula in the strong coupling case. In order to obtain the lineshape function $i\hbar\tilde{\Sigma}_{0f_i}(\bar{\omega})$, we must evaluate the quantities Δ_{0f_i} and ω_{1f_i} for both transition cases given in Eqs.(4.14) and (4.25), respectively:

$$\Delta_{0f_i} \equiv f_{1f_i}/j^+_{f_i} = -(L_T P^+_0 L_T j^+/\hbar^2)_{f_i}/j^+_{f_i}, \tag{5.8}$$

$$\omega_{1f_i} \equiv (L_T P^+_0 f_1/\hbar)_{f_i}/f_{1f_i} = -(L_T P^+_0 L_T P^+_0 L_T j^+/\hbar^3)_{f_i}/f_{1f_i}. \tag{5.9}$$

These quantities are contained in Eq.(4.45) and should be averaged over the impu rity configurations.

In the both transition cases, for the impurity scatterings we obtain

$$\Delta_{0\alpha_\nu\alpha_\nu} = -S_{11}/\hbar^2, \tag{5.10}$$

$$\omega_{1\alpha_\nu\alpha_\nu} = S_{12}/\hbar S_{11}, \tag{5.11}$$

$$\Delta_{0\alpha+1\alpha} = -S_{13}/\hbar^2, \tag{5.12}$$

$$\omega_{1\alpha+1\alpha} = S_{14}/\hbar S_{13}, \tag{5.13}$$

where

$$S_{11} \equiv < \sum_{\lambda(\neq\alpha)} [(h_{\epsilon-i})_{\alpha_\nu\lambda_\nu}\{(h_{\epsilon-i})_{\lambda_\nu\alpha_\nu} - (h_{\epsilon-i})_{\lambda_\nu\alpha_\nu} j^+_{\lambda\lambda_\nu}/j^+_{\alpha_\nu\alpha_\nu}\} + \{(h_{\epsilon-i})_{\alpha_\nu\lambda_\nu} - (h_{\epsilon-i})_{\alpha_\nu\lambda_\nu} j^+_{\lambda_\nu\lambda_\nu}/j^+_{\alpha_\nu\alpha_\nu}\}(h_{\epsilon-i})_{\lambda_\nu\alpha_\nu}] >_{imp}, \tag{5.14}$$

$$S_{12} \equiv < \sum_{\lambda(\neq\alpha)} [(h_{\epsilon-i})_{\alpha_\nu\lambda_\nu}(E^+_\lambda - E^+_\alpha) \times \{(h_{\epsilon-i})_{\lambda_\nu\alpha_\nu} - (h_{\epsilon-i})_{\lambda_\nu\alpha_\nu} j^+_{\lambda\lambda_\nu}/j^+_{\alpha_\nu\alpha_\nu}\} + (E^+_\alpha - E^+_\lambda)\{(h_{\epsilon-i})_{\alpha_\nu\lambda_\nu} - (h_{\epsilon-i})_{\alpha_\nu\lambda_\nu} j^+_{\lambda_\nu\alpha_\nu}/j^+_{\alpha_\nu\alpha_\nu}\}(h_{\epsilon-i})_{\lambda_\nu\alpha_\nu} + 2(h_{\epsilon-i})_{\alpha_\nu\lambda_\nu}\{(E^+_\alpha - E^+_\alpha) - (E^+_\lambda - E^+_\lambda)\} \times (h_{\epsilon-i})_{\lambda_\nu\alpha_\nu} j^+_{\lambda\lambda_\nu}/j^+_{\alpha_\nu\alpha_\nu} >_{imp}, \tag{5.15}$$

$$S_{13} \equiv < \sum_{\lambda(\neq\alpha+1)} (h_{\epsilon-i})_{\alpha+1\lambda}\{(h_{\epsilon-i})_{\lambda\alpha+1} - (h_{\epsilon-i})_{\lambda-1\alpha+1} j^+_{\lambda-1\lambda}/j^+_{\alpha+1\alpha}\} + \sum_{\lambda(\neq\alpha)} \{(h_{\epsilon-i})_{\alpha\lambda} - (h_{\epsilon-i})_{\alpha+1\lambda+1} j^+_{\lambda+1\lambda}/j^+_{\alpha+1\alpha}\}(h_{\epsilon-i})_{\lambda\alpha} >_{imp}, \tag{5.16}$$

$$S_{14} \equiv < \sum_{\lambda(\neq\alpha+1)} (E_\lambda - E_\alpha)(h_{\epsilon-i})_{\alpha+1\lambda} \times \{(h_{\epsilon-i})_{\lambda\alpha+1} - (h_{\epsilon-i})_{\lambda-1\alpha+1} j^+_{\lambda\lambda-1}/j^+_{\alpha+1\alpha}\} + \sum_{\lambda(\neq\alpha)} (E_{\alpha+1} - E_\lambda) \times \{(h_{\epsilon-i})_{\alpha\lambda} - (h_{\epsilon-i})_{\alpha+1\lambda+1} j^+_{\lambda+1\lambda}/j^+_{\alpha+1\alpha}\}(h_{\epsilon-i})_{\lambda\alpha} >_{imp}. \tag{5.17}$$

To derive Eqs(5.10), (5.11), (5.12), and (5.13) we have used Eqs.(5.3) and (5.4), and utilized the fact that any terms including odd number of $h_{\epsilon-i}$ disappear in the impurity average. Then, taking into account Eqs.(4.45) and (5.10)-(5.13) we obtain

$$i\hbar\tilde{\Sigma}_{0\alpha_\nu\alpha_\nu}(\bar{\omega}) = < \sum_{\lambda(\neq\alpha)} \left\{ \frac{(h_{\epsilon-i})_{\alpha_\nu\lambda_\nu}\{(h_{\epsilon-i})_{\lambda_\nu\alpha_\nu} - (h_{\epsilon-i})_{\lambda_\nu\alpha_\nu} j^+_{\lambda\lambda_\nu}/j^+_{\alpha_\nu\alpha_\nu}\}}{\hbar\bar{\omega} - E^+_\lambda + E^+_\alpha + \Xi_{11} - i\hbar\tilde{\Sigma}_{1\alpha_\nu\alpha_\nu}(\bar{\omega})} + \frac{\{(h_{\epsilon-i})_{\alpha_\nu\lambda_\nu} - (h_{\epsilon-i})_{\alpha_\nu\lambda_\nu} j^+_{\lambda\lambda_\nu}/j^+_{\alpha_\nu\alpha_\nu}\}(h_{\epsilon-i})_{\lambda_\nu\alpha_\nu}}{\hbar\bar{\omega} - E^+_\alpha + E^+_\lambda + \Xi_{12} - i\hbar\tilde{\Sigma}_{1\alpha_\nu\alpha_\nu}(\bar{\omega})} \right\} >_{imp},$$

(interband transition) (5.18)

$$i\hbar\tilde{\Sigma}_{0\alpha+1\alpha}(\bar{\omega}) = < \sum_{\lambda(\neq\alpha+1)} \frac{(h_{\epsilon-i})_{\alpha+1\lambda}\{(h_{\epsilon-i})_{\lambda\alpha+1} - (h_{\epsilon-i})_{\lambda-1\alpha} j^+_{\lambda\lambda-1}/j^+_{\alpha+1\alpha}\}}{\hbar\bar{\omega} - E_\lambda + E_\alpha + \Xi_{13} - i\hbar\tilde{\Sigma}_{1\alpha+1\alpha}(\bar{\omega})} + \sum_{\lambda(\neq\alpha)} \frac{\{(h_{\epsilon-i})_{\lambda\alpha+1} - (h_{\epsilon-i})_{\lambda-1\alpha} j^+_{\lambda\lambda-1}/j^+_{\alpha+1\alpha}\}(h_{\epsilon-i})_{\alpha+1\lambda}}{\hbar\bar{\omega} - E_{\alpha+1} + E_\lambda + \Xi_{14} - i\hbar\tilde{\Sigma}_{1\alpha+1\alpha}(\bar{\omega})} >_{imp},$$

(intraband transition) (5.19)

where $i\hbar\tilde{\Sigma}_{1f_i}(\bar{\omega})$ of Eqs.(5.26)and(5.27) in the denominator is the high-order colli sion term given in Eq.(4.45) and

$$\Xi_{11} = \{(E^+_\lambda - E^+_\alpha)S_{11} - S_{12}\}/S_{11}, \tag{5.20}$$

$$\Xi_{12} = \{(E^+_\alpha - E^+_\lambda)S_{12} - S_{13}\}/S_{11}, \tag{5.21}$$

$$\Xi_{13} = \{(E_\lambda - E_\alpha)S_{13} - S_{14}\}/S_{13}, \tag{5.22}$$

$$\Xi_{14} = \{(E_{\alpha+1} - E_\lambda)S_{13} - S_{14}\}/S_{13}. \tag{5.23}$$

Eqs.(5.18) and (5.19) are the general formula for the strongly interacting electron-impurity scattering case. If the high-order collision terms $(i\hbar\tilde{\Sigma}_{1\alpha_\nu\alpha_\nu}(\bar{\omega}), i\hbar\tilde{\Sigma}_{1\alpha+1\alpha}(\bar{\omega}))$ in the denominator of Eqs.(5.18) and (5.19) can also be ap-

proximated by the lineshape function ($i\hbar\tilde{\Sigma}_{0\alpha_e\alpha_e}(\bar{\omega}), i\hbar\tilde{\Sigma}_{0\alpha+1\alpha}(\bar{\omega})$) as on the lefthand side of the said equation , we can obtain an infinite number of coupled equations for the lineshape functions, which is similar to those of Shin et al.(1973) based on the Nakajima projection operator method(1958), of Lodder et al.(1968) based on the proper connected diagram approach, and of Prasad(1982) based on the coherent potential approximation approach. If the quantities Ξ_{i3}, Ξ_{i4} and the high-order collision term $i\hbar\tilde{\Sigma}_1(\bar{\omega})$ are neglected, Eq.(5.19) is reduced to Eq.(5.2) obtained up to the second order terms for the weak coupling case.

Real and imaginary parts of Eqs.(5.1), (5.2), (5.18) and (5.19) give the lineshift and linewidth, respectively. By using Eqs.(4.47), (4.48) and (4.49), the frequency-dependent conductivity is given by

$$Re\{\sigma_{\quad}(\omega)\} = \frac{N^2}{\Omega}\sum_{\alpha_e\alpha_e}\frac{f(E_\alpha^e)-f(E_\alpha^e)}{E_\alpha^e-E_\alpha^e}|j^*_{\alpha_e\alpha_e}|^2 \times\frac{\hat{\Gamma}_{0\alpha_e\alpha_e}(\omega)}{|\hbar\omega-E_\alpha^e+E_\alpha^e-\hbar\tilde{\nabla}_{0\alpha_e\alpha_e}(\omega)|^2+|\hbar\hat{\Gamma}_{0\alpha_e\alpha_e}(\omega)|^2} \quad \textit{(interband transition)} \quad (5.24)$$

$$Re\{\sigma_{+-}(\omega)\} = \frac{N^2}{\Omega}\sum_{\alpha}\frac{f(E_\alpha)-f(E_{\alpha+1})}{E_{\alpha+1}-E_\alpha}|j^*_{\alpha+1\alpha}|^2 \times\frac{\hat{\Gamma}_{0\alpha+1\alpha}(\omega)}{|\hbar\omega-E_{\alpha+1}+E_\alpha-\hbar\tilde{\nabla}_{0\alpha+1\alpha}(\omega)|^2+|\hbar\hat{\Gamma}_{0\alpha+1\alpha}(\omega)|^2} \quad \textit{(intraband transition)} \quad (5.25)$$

where Re means "the real part of". $\tilde{\Gamma}_{0\alpha_e\alpha_e}(\omega)$ and $\tilde{\nabla}_{0\alpha_e\alpha_e}(\omega)$ associated with the direct interband transition, respectively, can be calculated from Eqs.(5.1) and (5.7) for a weak coupling case. Futhermore, the quantities $\tilde{\Gamma}_{0\alpha+1\alpha}(\omega)$, $\tilde{\nabla}_{0\alpha+1\alpha}(\omega)$ related with the intraband transition can be evaluated from Eqs.(5.2) and (5.8) for a weak one as follows:

$$\begin{aligned}\hbar\hat{\Gamma}_{0\alpha_e\alpha_e}(\omega) &\equiv Im\{i\hbar\tilde{\Sigma}_{0\alpha_e\alpha_e}(\omega)\}\\ &= \pi<\sum_{\lambda(\neq\alpha)}|(h_{e-i})_{\alpha_e\lambda_e}\{(h_{e-i})_{\lambda\alpha_e}-(h_{e-i})_{\lambda\alpha_e}j^*_{\lambda\lambda_e}/j^*_{\alpha_e\alpha_e}\}\\ &\quad\times\delta(\hbar\omega-E_\lambda^e+E_\alpha^e)\\ &\quad+\{(h_{e-i})_{\alpha_e\lambda_e}-(h_{e-i})_{\alpha_e\lambda_e}j^*_{\lambda\lambda_e}/j^*_{\alpha_e\alpha_e}\}(h_{e-i})_{\lambda_e\alpha_e}\delta(\hbar\omega-E_\alpha^e+E_\lambda^e)\\ &\quad+\frac{(h_{e-i})_{\alpha_e\lambda_e}\{(E_\lambda^e-E_\alpha^e)-(E_\lambda^e-E_\alpha^e)\}(h_{e-i})_{\lambda_e\alpha_e}j^*_{\lambda\lambda_e}/j^*_{\alpha_e\alpha_e}}{E_\lambda^e-E_\alpha^e}\\ &\quad\times\{\delta(\hbar\omega-E_\lambda^e+E_\alpha^e)-\delta(\hbar\omega-E_\lambda^e+E_\lambda^e)\}\\ &\quad+\frac{(h_{e-i})_{\alpha_e\lambda_e}\{(E_\lambda^e-E_\alpha^e)-(E_\lambda^e-E_\alpha^e)\}(h_{e-i})_{\lambda_e\alpha_e}j^*_{\lambda\lambda_e}/j^*_{\alpha_e\alpha_e}}{E_\alpha^e-E_\lambda^e}\\ &\quad\times\{\delta(\hbar\omega-E_\alpha^e+E_\lambda^e)-\delta(\hbar\omega-E_\lambda^e+E_\lambda^e)\}|>_{imp}\end{aligned} \quad \textit{(interband transition)} \quad (5.26)$$

$$\begin{aligned}\hbar\hat{\Gamma}_{0\alpha+1\alpha}(\omega) &\equiv Im\{i\hbar\tilde{\Sigma}_{0\alpha+1\alpha}(\omega)\}\\ &= \pi<\sum_{\lambda(\neq\alpha+1)}(h_{e-i})_{\alpha+1\lambda}\{(h_{e-i})_{\lambda\alpha+1}-(h_{e-i})_{\lambda\alpha}j^*_{\lambda\lambda}/j^*_{\alpha+1\alpha}\}\\ &\quad\times\delta(\hbar\omega-E_\lambda+E_\alpha)\\ &\quad+\sum_{\lambda(\neq\alpha)}\{(h_{e-i})_{\alpha\lambda}-(h_{e-i})_{\alpha+1\lambda}j^*_{\lambda\lambda}/j^*_{\alpha+1\alpha}\}(h_{e-i})_{\lambda\alpha}\\ &\quad\times\delta(\hbar\omega-E_{\alpha+1}+E_\lambda)>_{imp}\end{aligned} \quad \textit{(intraband transition)} \quad (5.27)$$

for weak coupling. We then can calculate the lineshifts $\hbar\tilde{\nabla}_{0\alpha_e\alpha_e}(\omega)[\equiv Re\{i\hbar\tilde{\Sigma}_{0\alpha_e\alpha_e}(\omega)\}]$ and $\hbar\tilde{\nabla}_{0\alpha+1\alpha}(\omega)[\equiv Re\{i\hbar\tilde{\Sigma}_{0\alpha+1\alpha}(\omega)\}]$, respectively, through a Kramers-Kronig re lation:

$$\tilde{\nabla}_0(\omega)=\frac{1}{\pi}P\int_{-\infty}^{\infty}\frac{\tilde{\Gamma}_0(\omega')}{\omega-\omega'}d\omega' \quad (5.28)$$

where $\Gamma_0(\omega')$ is given by Eqs.(5.26) and (5.27). To obtain Eqs.(5.26) and (5.27), we have used the Dirac identity

$$\lim_{\varepsilon\to0^+}(x\pm i\varepsilon)^{-1}=P(1/x)\mp i\pi\delta(x), \quad (5.29)$$

where P denotes Cauchy's principle-value integral.

Considering Eqs.(5.18), and (5.19) for a strong coupling, we obtain the following equations for the spectral linewidths ($\hbar\tilde{\Gamma}_{0\alpha_e\alpha_e}(\omega)$, $\hbar\tilde{\Gamma}_{0\alpha+1\alpha}(\omega)$) and the lineshifts ($\hbar\tilde{\nabla}_{0\alpha_e\alpha_e}(\omega)$, $\hbar\tilde{\nabla}_{0\alpha+1\alpha}(\omega)$) associated with both the direct interband and the intra- band magneto-optical absorption:

$$\begin{aligned}\hbar\hat{\Gamma}_{0\alpha_e\alpha_e}(\omega) &=<\sum_{\lambda(\neq\alpha)}|\frac{(h_{e-i})_{\alpha_e\lambda_e}\{(h_{e-i})_{\lambda\alpha_e}-(h_{e-i})_{\lambda\alpha_e}j^*_{\lambda\lambda_e}/j^*_{\alpha_e\alpha_e}\}\hbar\hat{\Gamma}_{0\alpha_e\alpha_e}(\omega)}{|\hbar\omega-E_\lambda^e+E_\alpha^e+\Xi_{i1}+\hbar\tilde{\nabla}_{0\alpha_e\alpha_e}(\omega)|^2+|\hbar\hat{\Gamma}_{0\alpha_e\alpha_e}(\omega)|^2}\\ &\quad+\frac{\{(h_{e-i})_{\alpha_e\lambda_e}-(h_{e-i})_{\alpha_e\lambda_e}j^*_{\lambda\lambda_e}/j^*_{\alpha_e\alpha_e}\}(h_{e-i})_{\lambda_e\alpha_e}\hbar\hat{\Gamma}_{0\alpha_e\alpha_e}(\omega)}{|\hbar\omega-E_\alpha^e+E_\lambda^e+\Xi_{i2}+\hbar\tilde{\nabla}_{0\alpha_e\alpha_e}(\omega)|^2+|\hbar\hat{\Gamma}_{0\alpha_e\alpha_e}(\omega)|^2}|>_{imp}\end{aligned} \quad \textit{(interband transition)} \quad (5.30)$$

$$\begin{aligned}\hbar\hat{\Gamma}_{0\alpha+1\alpha}(\omega) &=<\sum_{\lambda(\neq\alpha+1)}\frac{(h_{e-i})_{\alpha+1\lambda}\{(h_{e-i})_{\lambda\alpha+1}-(h_{e-i})_{\lambda-1\alpha}j^*_{\lambda\lambda-1}/j^*_{\alpha+1\alpha}\}\hbar\hat{\Gamma}_{0\alpha+1\alpha}(\omega)}{|\hbar\omega-E_\lambda+E_\alpha+\Xi_{i3}+\hbar\tilde{\nabla}_{0\alpha+1\alpha}(\omega)|^2+|\hbar\hat{\Gamma}_{0\alpha+1\alpha}(\omega)|^2}\\ &\quad+\sum_{\lambda(\neq\alpha)}\frac{\{(h_{e-i})_{\alpha\lambda+1}-(h_{e-i})_{\lambda-1\alpha}j^*_{\lambda\lambda-1}/j^*_{\alpha+1\alpha}\}(h_{e-i})_{\alpha+1\lambda}\hbar\hat{\Gamma}_{0\alpha+1\alpha}(\omega)}{|\hbar\omega-E_{\alpha+1}+E_\lambda+\Xi_{i4}+\hbar\tilde{\nabla}_{0\alpha+1\alpha}(\omega)|^2+|\hbar\hat{\Gamma}_{0\alpha+1\alpha}(\omega)|^2}|>_{imp}\end{aligned} \quad \textit{(intraband transition)} \quad (5.31)$$

$$\begin{aligned}\hbar\tilde{\nabla}_{0\alpha_e\alpha_e}(\omega) &=<\sum_{\lambda(\neq\alpha)}|\frac{(h_{e-i})_{\alpha_e\lambda_e}\{(h_{e-i})_{\lambda\alpha_e}-(h_{e-i})_{\lambda\alpha_e}j^*_{\lambda\lambda_e}/j^*_{\alpha_e\alpha_e}\}}{|\hbar\omega-E_\lambda^e+E_\alpha^e+\Xi_{i1}+\hbar\tilde{\nabla}_{0\alpha_e\alpha_e}(\omega)|^2+|\hbar\hat{\Gamma}_{0\alpha_e\alpha_e}(\omega)|^2}\\ &\quad\times(\hbar\omega-E_\lambda^e+E_\alpha^e+\Xi_{i1})\end{aligned}$$

$$+\frac{\{(h_{\epsilon-i})_{\alpha,\lambda_\alpha}-(h_{\epsilon-i})_{\alpha,\lambda_\alpha}j^*_{\lambda_\alpha\lambda_\alpha}/j^*_{\alpha,\alpha}\}(h_{\epsilon-i})_{\lambda_\alpha\alpha}}{|\hbar\omega-E^*_\alpha+E^*_\lambda+\Xi_{i\beta}+\hbar\tilde{\nabla}_{1\alpha,\alpha}(\omega)|^2+|\hbar\tilde{\Gamma}_{1\alpha,\alpha}(\omega)|^2}$$
$$\times(\hbar\omega-E^*_\alpha+E^*_\lambda+\Xi_{i\beta})|>_{imp}$$

(*interband transition*) (5.32)

$$\hbar\tilde{\nabla}_{0\alpha+1\alpha}(\omega)$$
$$=<\sum_{\lambda(\neq\alpha+1)}\frac{(h_{\epsilon-i})_{\alpha+1\lambda}\{(h_{\epsilon-i})_{\lambda-1\alpha}j^*_{\lambda\lambda-1}/j^*_{\alpha+1\alpha}\}}{|\hbar\omega-E_\lambda+E_\alpha+\Xi_{i\beta}+\hbar\tilde{\nabla}_{1\alpha+1\alpha}(\omega)|^2+|\hbar\tilde{\Gamma}_{1\alpha+1\alpha}(\omega)|^2}$$
$$\times(\hbar\omega-E_\lambda+E_\alpha+\Xi_{i\beta})$$
$$+\sum_{\lambda(\neq\alpha)}\frac{\{(h_{\epsilon-i})_{\lambda\alpha+1}-(h_{\epsilon-i})_{\lambda-1\alpha}j^*_{\lambda\lambda-1}/j^*_{\alpha+1\alpha}\}(h_{\epsilon-i})_{\alpha+1\lambda}}{|\hbar\omega-E_{\alpha+1}+E_\lambda+\Xi_{i\beta}+\hbar\tilde{\nabla}_{1\alpha+1\alpha}(\omega)|^2+|\hbar\tilde{\Gamma}_{1\alpha+1\alpha}(\omega)|^2}$$
$$\times(\hbar\omega-E_{\alpha+1}+E_\lambda+\Xi_{i\beta})|>_{imp}$$

(*intraband transition*) (5.33)

where $\tilde{\nabla}_1(\omega)$ and $\tilde{\Gamma}_1(\omega)$ in Eqs.(5.30), (5.31), (5.32) and (5.33), respectively, are the real part and imaginary part of the high order self-energy $(-i\hbar\tilde{\Sigma}_{1fi}(\omega))$ in Eq.(4.45). If $\tilde{\Gamma}_1(\omega)$ and $\tilde{\nabla}_1(\omega)$ of Eqs.(5.30), (5.31), (5.32) and (5.33) for a strong coupling are approximated by $\tilde{\Gamma}_0(\omega)$ and $\tilde{\nabla}_0(\omega)$, respectively, we obtain an infinite number of coupled equations for the linewidths and the frequency shifts, which is similar to those of Suzuki The symbols *Re* and *Im* in Eqs.(5.26) and (5.32) denote, respectively, the real and the imaginary parts of the quantity. It should be noted that both of these quantities are functions of temperature, the external magnetic field, the impurity concentration and the incident photon frequency.

6. Conclusion

So far we have formulated the theory of magneto-optical absorption lineshape for both direct interband and intraband transitions due to the interaction with impurities in semiconductors.

The perturbation has been dealt with by the two techniques based on the Mori-type method of calculation. One is a closed-form representation which is applica- ble to the weak scattering case and the other is a continued-fraction form repre-sentation which is applicable to the strong scattering case. The continued-fraction representation is expressed by both the infinite expansion of the finite continued fraction order and the infinite continued fraction representation.

For sufficiently weak electron-impurity coupling, the results otained by the in-traband transition are identical with those of Argyres et al. and Kawabata However, in the case of the direct interband transition, our results for the second order of the scattering potential differ from those of Choi et al.(1984) and Yi et al.(1987). The main reason results in the approximation that L_T of Eq.(4.17) is replaced by L_{eB} under the assumption that $P'_0(L_e + L_B)X) = 0$ for any operator X. For strong electron-impurity coupling, the results obtained for the intraband transition are similar to those of some other authors(Prasad 1982 and Suzuki et al. 1982) ob-tained by using the renormalization of the superpropagators to include many body coherence effects in the cyclotron resonance transition problem. These results are given in the iterative manner while our result is given in the continued-fraction representation. The results obtained by the interband transition are similar to those of Suzuki et al.(1988) obtained by resolvent superoperator method. Thus we may claim that applying the Mori-type projection approach we can improve and gener-alize the earlier theories(Prasad 1982, Ryu et al. 1984, Suzuki et al. 1982, 1988) for the collision broadening effect as well as the shift in the magneto-optical transitions at finite temperatures.

There are several important issues including nonparabolicity, degenerate band, spin and spin-orbit coupling effects, valence band mixing, and the interaction be-tween electrons and holes (exitons) which would be important at low temperatures. All these works are left for future studies.

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References

Argyres P.N. and J.L. Sigel, 1974. Theory of cyclotron resonance absorption, *Phys. Rev.* B10, 1139-1148; 1984. *Solid State Commun.*, 50, 279-281.

Burstein E, G.S. Picus, R.F Wallis and F.

Blatt, 1959. Zeeman-Type Magneto-Optical Studies of Interband Transition in Semiconductors, *Phys. Rev.* 113, 15-33.

Ciobanu G. and L. Banyai, 1963. On the kinetic theory of magneto-optical phenomena by Green function method, *Phys. Stat. Sol.* 3, 2299-2304.

Choi S.D., and O.H. Chung, 1983. Theory of cyclotron resonance lineshape electron-phonon system in two coupling shcemes, *Solid State Commun.*, 46, 717-720; 1984. *J. Phys. Chem. Sol.* 45, 1243-1247.

Fujita S., 1966. Introduction to nonequil ibrium statistical quantum mechanics, (Saunders, Philadelphia,), Chapter 6.

Kawabata A., 1967. Theory of cyclotron resonance line width, *J. Phys. Soc. Jpn.*, 23, 999-1005.

Kobori H., T. Ohyama, and E. Otsuka, 1987. Quantum limit cyclotron resonance in *p*-GaAs, *Solid State Commun.* 63, 123-126; 1990. *J. Phys. Soc. Jpn.* 59, 2141-2177.

Kubo R., 1957. Statistical mechanical theory of irreversible processes. I., *J. Phys. Soc. Jpn.*, 12, 570-586.

Lodder A., and S. Fujita, 1968. Theory of cyclotron resonance width, *J. Phys. Soc. Jpn.* 25, 774-783.

Lynch D.W., 1985. Handbook of Optical Constants of Solids, ed. E.D. Palik (Academic Press, New York,), Chapt. 10.

Madelung O., 1987. Introduction to Solid State Theory, (Springer-Verlag, Berlin).

Matsuda O., and E. Otsuka, 1979. Cyclotron resonance study of conduction electron in *n*-type indium antimonide under a strong magnetic field-I., *J. Phys. Chem. Solids* 40, 809-817.

McCombe B.D, R. Kaplan, R.J. Wagner, E. Gornik, and W. Muller, 1976. Absoption and emission studies of the quantumlimit cyclotron line width in *n*-InSb, *Phys. Rev.* B13, 2536-2539.

Mori H., 1965. Transport, Collective Motion, and Brownian Motion, Progr. Theor. Phys. 33, 423-455.

Nakajima S., 1958. On quantum theory of transport phenomena, *Progr. Theor. Phys.* 20, 948-959.

Prasad M., 1982. Quantum-statistical theories of the cyclotron resonance lineshape in parabolic semiconductors, *Phys. Stat. Sol.* B109, 11-34.

Royen J.V., J. De Sitter, J.T. Devreese, 1984. Calculation of the cyclotron-resonance spectrum in the quantum limit for seconductors, *Phys. Rev.* B30, 7154-7169.

Ryu J.Y. and S.D Choi, 1984. Cyclotron resonance lineshape function based on methods of Mori and Kawabata, *Progr. Thoer. Phys* 72, 429-440; 1985. *Phys. Rev.* B32, 7769-7775; 1991. *Phys. Rev.* B44, 11329-11338.

Shin E.E., P.N Argyres and B. Lax, 1973. High-field anisotropic shielding and cyclotron-resonance linewidth in the quantum limit, *Phys. Rev.* B7, 5408-5410.

Suzuki A., 1988. Theory of phonon-induced interband magnetooptical absorption lineshape, *Progr. Theor. Phys.* 79, 343-358.

Suzuki A., and D. Dunn, 1982. Quantum-limit cyclotron resonance linewidth due to an electron-phonon interaction, *Phys. Rev.* B25, 7754-7764; 1986. *Phys. Rev.* B33, 1047-1057.

Yi S.N., J.Y. Ryu, O.H. Chung, Y.C. Sug and S.D Choi, 1987. Theory of Direct-Interband-Transition Lineshapes Based on Mori's method, I Nuovo Cimento 9D, 927-940.

〈국문초록〉

저밀도 불순물에 대한 자기 광흡수 선모양함수 이론

Kubo의 선형 응답이론을 토대로 전자-불순물계에 대한 자기 광전이 이론을 Faraday 조건에서 제시하였다. 계의 진동수 의존 전기전도도는 Mori형태의 사영연산자 방법을 사용하여 계산하였고, 포물선 밴드 모델에서 전자와 불순물 사이에 약한 상호작용하는 경우와 강한 상호작용하는 경우에 적용할 수 있는 일반적인 형태의 선모양함수를 각각 두가지 다른 방법으로 제시하였다. 직접 대간 전이 및 대내 전이에 대한 선모양 함수가 온도, 자기장, 불순물 농도 및 입사광 진동수의 함수로 나타남을 볼 수 있고 그 결과를 타이론과 비교하였다.