A Study on a Theory of Shear Spinning for Plastic Deformation of Metals

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Summary

Spinning is a more recent manufacturing technique which is a development of the metal working process widely used to fabricate sheet metal components having rotational symetry.

Using a mandrel cone angle and a roller having a specific profile, cones were produced from commercially pure aluminum sheet, to examine the effects of mandrel speed, feed rate and fractional reduction in wall thickness of the formed conical work piece on the component forces during spin-forging operation.

Analytical expressions for shear-spinning stresses and tangential force are suggested on the basis of the combined rolling and extrusion of the metal.

On the basis of the foregoing elementary considerations, values for the maximum theoretical reduction are derived for an ideally plastic material and for a typical strain-hardening material,

A three component force dynamometer was measured the tangential, radial and axial components of the resultant force exerted at the work piece-roller interface.

The efficiency of the deformation is invastigated on the basis of experimental values of the tangential force.

Intrnduction

In the working process, The machine may be either horibontal or vertical.

The components are the head-stock, the tailstock, the longitudinal slides, and the cross slides.

the cross slides are affixed the rollers which cause the material to take the shape of rotating mandrel.

The longtitudinal slides feed the cross slides along the length of the mandrel.

The cross slides follow the control of the mandrel.

The main slides and cross slides are hydraulically actuated.

Owing to the considerable forces necessary to spin some material, mechanical drives using lead-scews or racks and pinions are not practical.

Our standard machines have a force on the cross slide of 50,000Ib.

The main slide resists a force of 50,000Ib and the tailstock a force of 30,000Ib.

The total load against the head-stock and its bearing is 130,000lb.

The maxinum speed of the head-stock is approximately 600rpm.

The rollers and tool rings are driven by contact with the work and the mandrel.

Standard machines are made to spin parts 50in long and 42in diameter.

General Description

In spining operation, as they are performed recently, this means the following things;

(1) The tool can be longer be manipulated back and forth, but must perform the deformation in one pass.

(2) The rotational speed, the feed, and the head in pressure have to be fixed and preset before spining is started.

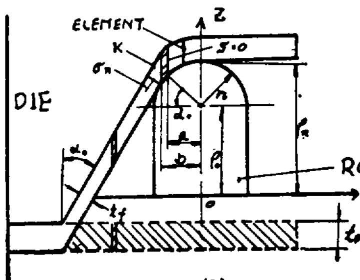
In this analysis, the geometry of the operation have been described mathematically.

The equations of the cone and the roller have also been formulated and the boundaries of the area of contact between the roller and the cone have then been found.

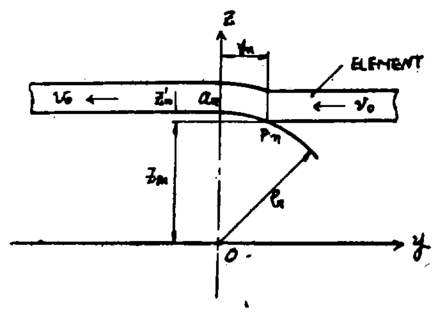
A solution has been derived for the plastic work of deformation which was based on the deformation theory (Stress-Strain Law) and the solution was computed for the following material.

(1) Homogeneous and isotropic material:

(2) Incompressible material:



(a)



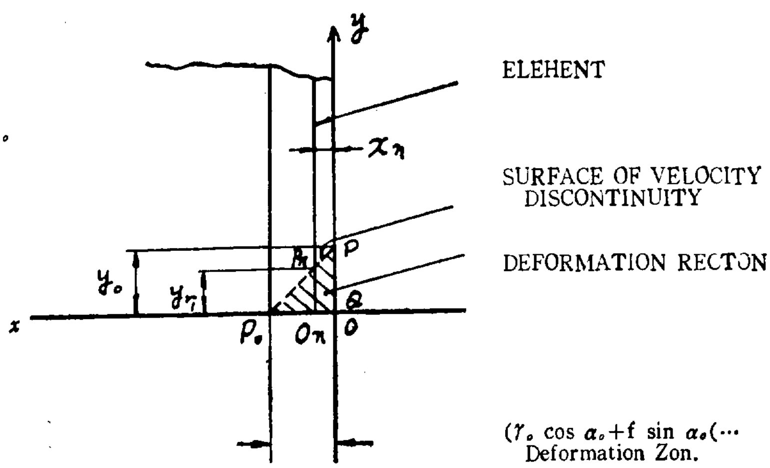
(b)

$a = -\gamma_o \cos \alpha_o$

$b = -\gamma_o \cos \alpha_o - f \sin \alpha_o$

$b - a = -f \sin \alpha_o$

$a \geqq x \geqq b$ ···Final Stage of Deformation



(c)

Fig 1. Deformation in process.

Strain Rates ; The Strain Rates in a Cartesian Co-ordinates System Assume the Form ;

After finishing one complete rolling pass, the roller feeds $f\ sin\alpha_o$ to X-direction and $f\text{-}cos\alpha_o$ to z-direction.

This demands that the initial bank thickness t_o and the final thickness of the channel t_f for the slope angle α_o be related by the equation.

$$t_f = t_o\ sin\alpha_o \quad \cdots\cdots(1)$$

The relationship for the variation of the thickness is the same as to the shear spinning of cones.

Sometimes it is found convenient to consider half the engineering shear strains particulary when using tensor notation : then

$$\left.\begin{array}{l} \dot{E}_{xx}=\frac{\partial V_x}{\partial x} \quad \dot{E}_{yy}=\frac{\partial V_y}{\partial y} \quad \dot{E}_{zz}=\frac{\partial V_z}{\partial z} \\ \dot{E}_{xy}=\frac{1}{2}\left(\frac{\partial V_y}{\partial x}+\frac{\partial V_x}{\partial y}\right) \\ \dot{E}_{yz}=\frac{1}{2}\left(\frac{\partial V_z}{\partial y}+\frac{\partial V_y}{\partial z}\right) \\ \dot{E}_{xz}=\frac{1}{2}\left(\frac{\partial V_x}{\partial z}+\frac{\partial V_z}{\partial x}\right) \end{array}\right\} \cdots\cdots(2)$$

From the defrmation mode, one gets the velocity in x and y direction.

$$\left.\begin{array}{l} V_x=0 \quad \left(=\frac{dx}{dt}\right) \\ V_y=-V_o \quad \left(=\frac{dy}{dt}\right) \end{array}\right. \cdots\cdots(3)$$

The geometry of the roller Zs can be described as [See Fig 1. (a)]

$$Z_s^2=\left[\sqrt{\gamma_o^2-X^2}+\rho_o^2\right]^2-y^2 \quad \cdots\cdots(4)$$

$$\text{Thus}\quad V_z=\frac{dZ_s}{dt}=\frac{\partial Z_s}{\partial x}\frac{dx}{dt}+\frac{\partial Z_s}{\partial y}\frac{dy}{dt}$$

$$=-V_o\frac{\partial Z_s}{\partial y} \quad \cdots\cdots(5)$$

Inserting the Value of Vx, Vy, Vz from equation (3) and (5) in to equation (2), one gets the following strain-rate fields.

$$\left.\begin{array}{l} \dot{E}_{yz}=\frac{1}{2}\frac{\partial V_z}{\partial y}=-\frac{V_o}{2}\frac{\partial^2 Z_s}{\partial y^2} \\ \dot{E}_{xz}=\frac{1}{2}\frac{\partial V_z}{\partial x}=-\frac{V_o}{2}\frac{\partial^2 Zs}{\partial x\partial y} \\ \text{all other } \dot{E}_{ij}=0 \end{array}\right\} \cdots\cdots(6)$$

Calculating the derivatives of Z_s from equation (4), one gets

$$\begin{array}{l} \frac{\partial Z_s}{\partial x}=-\frac{X\sqrt{y^2+Z_s^2}}{Z_s\sqrt{\gamma_o^2-X^2}} \\ \frac{\partial Z_s}{\partial y}=-\frac{y}{Z_s} \\ \frac{\partial^2 Z_s}{\partial x\partial y}=-\frac{(y^2+Z_s^2)\times\sqrt{y^2+Z_s^2}}{Z_s^3 y\sqrt{\gamma_o^2-X^2}} \\ \frac{\partial^2 Z_s}{\partial y^2}=-\frac{(y^2+Z_s^2)}{Z_s^3} \end{array} \quad \cdots\cdots(7)$$

The Power

The rate of total work done on metal under the deformation zone becomes

$$\dot{W}=\int_{vol}\frac{\partial_o}{\sqrt{3}}\sqrt{\left(\frac{\partial V_z}{\partial x}\right)^2+\left(\frac{\partial V_z}{\partial y}\right)^2}\,dv$$

$$+\int_s\frac{\partial_o}{\sqrt{3}}\triangle V_z\,ds \quad \cdots\cdots(8)$$

σ_o ; Effective stress

the first term ($\dot{W}_E$) ; the internal power of deformation the second term ($\dot{W}_S$) ; the shear loss over surfase of velocity of discontinuity $\triangle V_z$

$$\dot{w}=\frac{\partial_o V_o}{\sqrt{3}}\int_{P\,P_oQ}^{\text{surface area}}\left[\int_{Z=Z_s+t_o}^{Z=Z_s}\sqrt{\left(\frac{\partial^2 Z_s}{\partial x\partial y}\right)^2+\left(\frac{\partial^2 Z_s}{\partial y^2}\right)^2}\,dz\right]ds$$

$$+\int_{\substack{\text{line}\\ PP_nP_o}}\left[\int_{Z=Z_s+t_o}^{Z=Z_s}-\frac{\partial Z_s}{\partial y}dz\right]\sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2}\,dx$$

$$=\frac{\sigma_o V_o t_o}{\sqrt{3}}\int_X\int_Y\frac{\partial^2 Z_s}{\partial x\partial y}\sqrt{1+\delta^2}\;dx\cdot dy$$

$$+\int_X-\frac{\partial Z_s}{\partial y}\sqrt{1+\mu^2}\,dx \quad \cdots\cdots(9)$$

Where $\delta=\frac{\partial^2 Z_s}{\partial y^2}\Big/\frac{\partial^2 Z_s}{\partial x\partial y}=\frac{y\sqrt{\gamma_o^2-X^2}}{X\sqrt{y^2+Z_s^2}}=\frac{\partial x}{\partial y}$

on area PP_oQ $\cdots\cdots(10)$

$\mu=\frac{\partial y}{\partial x}$ on line $PP_n\rho_o=\frac{\partial y_n}{\partial x}$

δ and μ : constant during integration. the equation (8) can be written as.

$$\dot{w}=\frac{\partial_o V_o t_o}{\sqrt{3}}\left[\sqrt{1+\delta^2}\int_X\int_Y\frac{\partial^2 Z_s}{\partial x\partial y}dxdy+\sqrt{1+\mu^2}\int_X-\frac{\partial Z_s}{\partial y}dx\right] \quad \cdots\cdots(11)$$

$$=\frac{\partial_o V_o t_o}{\sqrt{3}}\left[\sqrt{1+\delta^2}\int_Y\frac{\partial Z_s}{\partial y}\Big|_{X=\text{lower limit}}^{X=\text{upperl imit}}dy+\sqrt{1+\mu^2}\int_X\frac{y_n}{Z_{sn}}dx\right]\doteqdot\frac{\partial_o V_o t_o}{\sqrt{3}}\left[\sqrt{1+\delta^2}\right.$$

$$\left\{Z'_n\Big|_{x=0}-Z_n'\Big|_{x=-\gamma\cos\alpha_o-f\sin\alpha_o}\right\}$$

$$\left.+\sqrt{1+\mu^2}\int_X\frac{2Z_n'}{y_n}dx\right]\doteqdot\frac{\partial_o V_o t_o}{3}\left[\sqrt{1+\delta^2}f\cos\alpha_o\right.$$

$$\left.+\sqrt{1+\mu^2}\left(\frac{2Z_n'}{y_n}\right)_{\text{ave}}(\gamma_o\cos\alpha_o+f\sin\alpha_o)\right] \cdots(12)$$

$$\left.\frac{2Z_n'}{y_n}\right|_{\text{ave}}=\frac{f\sin\alpha_o}{\gamma_o\cos\alpha_o+f\sin\alpha_o}\cdot\frac{2\gamma_o(\frac{1}{\sin\alpha_o}-1)}{(y_n)\text{ave}} \quad \cdots\cdots(13)$$

Insterting equation (13) into equation (12), one gets.

$$\dot{W}=\frac{\partial_o V_o t_o}{\sqrt{3}}\left[\sqrt{1+\delta^2}f\cos\alpha_o+\sqrt{1+\mu^2}\,f\sin\alpha_o\frac{2\gamma_o(\frac{1}{\sin\alpha_o}-1)}{(y_n)\text{ave}}\right]$$

$$=\frac{\partial_o V_o t_o}{\sqrt{3}}f\cos\alpha_o\left[\sqrt{1+\delta^2}+2\sqrt{1+\mu^2}\,\gamma_o\frac{(1-\sin\alpha_o)}{\cos\alpha_o(y_n)\text{are}}\right] \quad \cdots\cdots(14)$$

$$\dot{W}_{E.yz}=\frac{\partial_o V_o t_o}{\sqrt{3}}\int_X\int_Y-\frac{\partial^2 Z_s}{\partial y^2}dydx$$

$$=\frac{\partial_o V_o t_o}{\sqrt{3}}\int_X-\frac{\partial Z_s}{\partial y}\Big|_{y=0}^{y=y_n}dx$$

$$=\frac{\partial_o V_o t_o}{\sqrt{3}}\int_X\frac{\partial Z_s}{\partial y}\Big|_{y=y_n}dx \quad \cdots\cdots(15)$$

$$\dot{W}_s=\frac{\partial_o V_o t_o}{\sqrt{3}}\sqrt{1+\mu^2}\int_X-\frac{\partial Z_s}{\partial y}\Big|_{y=y_n}dx \quad \cdots\cdots(16)$$

comparing equation (15) with equation (16), on gets

$$\dot{W}_{Eyz}=\frac{\dot{W}_s}{\sqrt{1+\mu^2}} \quad \cdots\cdots(17)$$

As an approximation to δ, one gets

$$\delta=\frac{2\gamma_o(1-\sin\alpha_o)}{\cos\alpha_o(y_n)\text{ave}} \quad \cdots\cdots(18)$$

$$Z_s^2=\left[\gamma_o\sqrt{1-(\frac{X}{\gamma_o})^2}+\rho_o\right]^2-y^2$$

$$(Z_s+f\cos\alpha_o)^2=\left[\gamma_o\sqrt{1-(\frac{x+f\sin\alpha_o}{\gamma_o})^2}+\rho\right]^2 \quad \cdots\cdots(19)$$

From equation (19), one gets

$$Z_s^2+2f\cos\alpha_o\,Z_s+f^2\cos^2\alpha_o\doteqdot\left[\gamma_o\sqrt{1-(\frac{x}{\gamma_o})^2}+\rho_o\right]^2$$

$$-2\left[\gamma_o\sqrt{1-\left(\frac{X}{\gamma_o}\right)^2}+\rho_o\right]$$

$$\cdot\frac{f\sin\alpha_o\left(X+\frac{f\sin\alpha_o}{2}\right)}{\gamma_o}\quad\cdots\cdots\cdots\cdots(20)$$

Insering equation (20) to equation (4), oue gets

$$y_n^2\doteqdot 2f\cos\alpha_o\left\{\gamma_o\left[1-\left(\frac{1}{2}\left(\frac{x+f\sin\alpha_o}{\gamma_o}\right)^2\right)\right]\right.$$

$$\left.+\rho_o-\frac{f\cos\alpha_o}{2}\right\}+2\left\{\gamma_o\left[1-\frac{1}{2}\left(\frac{X}{\gamma_o}\right)^2\right]\right.$$

$$\left.+\rho_o\right\}\frac{f\sin\alpha_o\left(x+\frac{f\cos\alpha_o}{2}\right)}{\gamma_o}\quad\cdots\cdots\cdots(21)$$

$$(y_n)\text{ave}=y_n\Big|_{x=-\frac{(\gamma_o\cos\alpha_o+f\sin\alpha_o)}{2}}$$

$$=\left\langle 2f\cos\alpha_o\left\{\left[\gamma_o-\frac{(-\gamma_o\cos\alpha_o-f\sin\alpha_o)^2}{8\gamma_o}\right.\right.\right.$$

$$\left.\left.\left.+\rho_o\right]\left(1-\frac{\sin\alpha_o}{2}\right)-\frac{f\cos\alpha_o}{2}\right\}\right\rangle^{\frac{1}{2}}$$

$$\left(\frac{\partial y_n}{\partial x}\right)_{\text{ave}}=\frac{\partial y_n}{\partial x}\Big|_{x=-\frac{(\gamma_o\cos\alpha_o+f\sin\alpha_o)}{2}}$$

$$=\frac{f\cos\alpha_o)\left\langle\frac{\gamma_o\cos\alpha_o-f\sin\alpha_o}{2\gamma_o}\right.}{y_n}$$

$$\underline{+\tan\alpha_o\left\{1-\frac{(\gamma_o\cos\alpha_o+f\sin\alpha_o)^2}{8\gamma_o^2}+\frac{\rho_o}{\gamma_o}\right.}$$

$$\underline{\left.\left.-\frac{\gamma_o\cos\alpha_o(\gamma_o\cos\alpha_o+f\sin\alpha_o)}{4\gamma_o^2}\right\}\right\rangle}\quad\cdots\cdots(22)$$

$$\therefore\ \dot{W}=\dot{W}_E+\dot{W}_S=\frac{\partial_o V_o t_o}{3}f\cos\alpha_o\left[\sqrt{1+\delta^2}\right.$$

$$\left.+\delta\sqrt{1+\mu^2}\right]\quad\cdots\cdots\cdots\cdots\cdots\cdots(23)$$

Where $\mu=\left(\frac{\partial y_n}{\partial x}\right)_{\text{ave}}$ from equation (22)

$\delta=\left(\frac{\partial x}{\partial y}\right)_{\text{ave}}$ from equation (18)

Result and Discussion

In general, the power is the sum of the sum of the forces by the velocities directed, These results indicate that the three component forces during the spn-forging operations vary as the feed rate. In particular, the tangential component varies linearly with the feed rate whilst the radial and axial components do not.

The effects of each of the paremeters, feed f, cone inculuded half angle α_o, effective stress σ_o, μ from equation(22), δ from equation(22) on the tangential force, are given.

From there five parameters, one finds the value of the weighted tangential value.

For the analytical study, a displacment fixed was postulated, which gave the strain-rates field.

The strain-rates field satisfies automatically the compatibility conditions. From the power equation (23), we find some difference in solution.

Comparing the power equation(23) with the power for cone spinning from An upper-bound approach (Kim's theoritical resutls), the power equation(23) gives good approximation to actual power.

So we can expects that equation(23) gives us an upper-bound for the power of shear rolling of channel shape.

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국 문 초 록

소성변형시 전단스피닝 이론에 대한 연구

이 연구는 소성변형시 수식적인 해석과 원추체 가공시 정확한 동력을 구해 보려고 시도했다. 필자의 새로운 모델을 가정하여 소성변형영역을 해석하기 위한 상계법에서 주어진 결과와 또 실험치와 비교하니, 접선 방향의 동력계산식이 거의 일치가 됨을 입증된다. 원추가공 기계설계의 동력계산에 활용되리라 사료된다.