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First-Order Eikonal Correction to Optical Potential by Inverse Method in $^{12}C + {}^{12}C$ Elastic Scattering at $E_{lab} = 2400$ MeV.

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By solving the inverse scattering problem using the relation between the McIntyre phase shift and Woods-Saxon type optical potential, we calculated the optical potential for the $^{12}C + {}^{12}C$ elastic scattering at E_{lab}= 2400 MeV. The effect of the first-order eikonal correction to the heavy-ion optical potential is investigated. We found that this effect is significant around the surface region for the real potential in this scattering system.

I. INTRODUCTION

Heavy-ion elastic scattering has been studied by a variety of theoretical model. Strong absorption model (SAM) has been used extensively as a useful theoretical approach in the analysis of heavy-ion elastic scattering. In recent year, many theoretical investigations[1–3] were made in describing the elastic and inelastic scattering processes between heavy ions within the McIntyre SAM[4]. This model was known to provide a good description of nucleus-nucleus elastic scattering data over a wide energy range.

In most of studies of heavy-ion collisions, the central feature is the optical potential. The connection between the optical potentials and data (differential cross sections primarily) is the scattering matrix. Recently, there are several attempts[5–8] to evaluate optical potential from parametrized phase shift. These approaches are to employ a phase shift obtained by fitting the elastic scattering angular distribution data, in the SAM, to determine the heavy-ion optical potential by solving the inverse scattering problem. A practical solution of the inversion problems by using the quasi-classical limit of the high-energy approximation was reported[5]. By solving the inverse scattering problem at high energy, Fayyad *et al.*[6] obtained a Woods-Saxon type model potentials with the parameters that can be determined directly from the McIntyre parametrization of the phase shift, for heavy-ion elastic scattering. The inversion procedure[7] applied to Ericson's parametrized phase shifts was also performed by using the Glauber's approximation and was used to deduce local potentials describing the angular distributions of the elastic scattering of $^{12}C+{}^{12}C$ systems at diverse energies. Ahmad *et al.*[8] studied the effect of the non-eikonal corrections on the determination of heavy-ion optical potential from the diffraction model phase shifts at intermediate energies.

In previous papers[9–11], a phase shift analysis based on McIntyre SAM was presented. It was applied satisfactorily to the so-called Fresnel patterns $^{12}C+{}^{90}Zr$ and $^{12}C+{}^{208}Pb$ elastic scatterings at E_{lab}/A=35 MeV/nucleon[9]. In recent, elastic scattering data of $K^+ + {}^{12}C$ and $K^+ + {}^{40}Ca$ at 800 MeV/c are reproduced and inverse potentials are predicted[12] in the framework of McIntyre SAM.

In this paper, we will calculate the heavy-ion optical potential in the $^{12}C+{}^{12}C$ system at 2400 MeV by solving the inverse scattering problem. The calculated optical potential will be compared with the result of optical model analysis[13]. Furthermore, the effect of the first-order eikonal correction on heavy-ion optical potential by inversion will be investigated. In section II, we present the inverse solution to heavy-ion optical potential in detail. We present the first-order eikonal correction to optical potential by inversion in section III. Results and conclusions are presented in section IV.

II. INVERSE SOLUTION TO HEAVY-ION OPTICAL POTENTIAL

The elastic scattering amplitude for spin-zero particle via Coulomb and short-range central force is given by

$$f(\theta) = f_R(\theta) + \frac{1}{ik}\sum_{l=0}^{\infty}(l+\frac{1}{2})\exp(2i\sigma_l) \times (S_l^N - 1)P_l(\cos\theta). \quad (1)$$

Here $f_R(\theta)$ is the usual Rutherford scattering amplitude, k is the wave number and σ_l denotes the Coulomb phase shifts. The nuclear S-matrix, S_l^N, can be obtained from the nuclear phase shift $\chi(l)$ given by

$$S_l^N = \exp[i\chi(l)] = \exp[i\{\chi_R(l) + i\chi_I(l)\}]. \quad (2)$$

In this work, we use the McIntyre parametrization [4] of the S-matrix. The real and imaginary parts of the nuclear phase shift for McIntyre parametrization [4] are expressed

$$\chi_R(l) = 2\mu\{1 + \exp[(l - l_{g'})/\Delta_{g'}]\}^{-1} \quad (3)$$

and

$$\chi_I(l) = \ln[1 + \exp\{(l_g - l)/\Delta_g\}], \quad (4)$$

where $l_{g'}$ and $\Delta_{g'}$ are the grazing angular momentum and its width for $\chi_R(l)$, while l_g and Δ_g are the quantities for $\chi_I(l)$. And μ is the McIntyre phase shift parameter related with the strength of nuclear phase shift. The real and imaginary parts of nuclear phase shift $\chi(b)$ can further be written in terms of impact parameter as

$$\chi_R(b) = \frac{2\mu}{1 + \exp[(b - b_0')/d']} \quad (5)$$

and

$$\chi_I(b) = \ln\left[1 + \exp\left(\frac{b_0 - b}{d}\right)\right], \quad (6)$$

where b, b_0 and b_0' are the impact parameters normally given by $kb = l + \frac{1}{2}$, $kb_0 = l_g + \frac{1}{2}$ and $kb_0' = l_{g'} + \frac{1}{2}$, respectively. The diffusivity quantities d and d' are given by $d = \Delta_g/k$ and $d' = \Delta_{g'}/k$.

Using the relation between nuclear phase shift and optical potential

$$\chi(b) = -\frac{k}{E}\int_b^\infty \frac{r(\widetilde{V}(r) + i\widetilde{W}(r))}{\sqrt{r^2 - b^2}}dr, \quad (7)$$

we can write real and imaginary nuclear phase shifts as

$$\chi_R(b) = -\frac{k}{E}\int_b^\infty \frac{r\widetilde{V}(r)}{\sqrt{r^2 - b^2}}dr \quad (8)$$

and

$$\chi_I(b) = -\frac{k}{E}\int_b^\infty \frac{r\widetilde{W}(r)}{\sqrt{r^2 - b^2}}dr. \quad (9)$$

Eqs. (8) and (9) are of Abel's type[6] and the inverse solutions of these equations are given by

$$\widetilde{V}(r) = \frac{2E}{k\pi}\frac{1}{r}\frac{d}{dr}\int_r^\infty \frac{\chi_R(b)}{\sqrt{b^2 - r^2}}\, bdb \quad (10)$$

and

$$\widetilde{W}(r) = \frac{2E}{k\pi}\frac{1}{r}\frac{d}{dr}\int_r^\infty \frac{\chi_I(b)}{\sqrt{b^2 - r^2}}\, bdb. \quad (11)$$

Approximating the phase shift $\chi_R(b)$ and $\chi_I(b)$ in terms of sums of Gaussian shape and inserting this approximated phase shift forms into Eqs. (10) and (11), we get

$$\widetilde{V}(r) = -\frac{4\mu E}{\pi k\alpha}\sum_{n=1}^{N} c_n\sqrt{\pi n}\exp\left(-\frac{nr^2}{\alpha^2}\right) \quad (12)$$

and

$$\widetilde{W}(r) = -\frac{2E}{\pi k\beta}\sum_{n=1}^{N} b_n\sqrt{\pi n}\exp\left(-\frac{nr^2}{\beta^2}\right). \quad (13)$$

In order to relate above $\widetilde{V}(r)$ and $\widetilde{W}(r)$ with the familiar Woods-Saxon forms, let us rewrite $\widetilde{V}(r)$ and $\widetilde{W}(r)$ in the forms

$$\widetilde{V}(r) = -\frac{4\mu E}{\pi k\alpha}\frac{1}{1 + \exp[(r - R')/\Delta']} \quad (14)$$

and

$$\widetilde{W}(r) = -\frac{2E}{\pi k\beta}\frac{1}{1 + \exp[(r - R)/\Delta]}. \quad (15)$$

With the above parametrization, as is shown in Ref.[6], the following relations between the parameters of the corresponding phase shifts will hold. For the real part,

$$\frac{I_4(R',\Delta')}{I_2(R',\Delta')}=\frac{3}{2}\frac{I_3(b_0',d')}{I_1(b_0',d')} \tag{16}$$

and

$$\frac{I_6(R',\Delta')}{I_4(R',\Delta')}=\frac{5}{4}\frac{I_5(b_0',d')}{I_3(b_0',d')}. \tag{17}$$

For the imaginary part,

$$\frac{I_4(R,\Delta)}{I_2(R,\Delta)}=\frac{3}{4}\frac{I_4(b_0,d)}{I_2(b_0,d)} \tag{18}$$

and

$$\frac{I_6(R,\Delta)}{I_4(R,\Delta)}=\frac{5}{6}\frac{I_6(b_0,d)}{I_4(b_0,d)}, \tag{19}$$

where $I_\nu(x_0,a_0)$ is an integral given by

$$I_\nu(x_0,a_0)=\int_0^\infty \frac{x^\nu}{1+\exp[(x-x_0)/a_0]}\,dx. \tag{20}$$

By using a certain iteration procedure such as the Newton method[14], we can obtain parameters R' and Δ' from solving two nonlinear simultaneous Eqs. (16)-(17) and R and Δ from Eqs. (18)-(19), respectively.

The parameters α and β in Eqs. (14) and (15) are given by the formulae

$$\alpha=\frac{2}{\pi}[1+\exp(-b_0'/d')]I_0(R',\Delta') \tag{21}$$

and

$$\beta=\frac{2}{\pi}\frac{I_0(R,\Delta)}{\ln[1+\exp(b_0/d)]}, \tag{22}$$

where

$$I_0(R',\Delta')=R'+\Delta'\ln[1+\exp(-R'/\Delta')], \tag{23}$$

and $I_0(R,\Delta)$ in Eq.(22) is given by a similar expression in R and Δ.

III. FIRST-ORDER EIKONAL CORRECTION TO OPTICAL POTENTIAL BY INVERSION

The phase shift is identified with the zeroth-order eikonal phase shift function, which is related to the heavy ion optical model potential $U_{op}(r)=V(r)+iW(r)$. Wallace [15, 16] expanded the WKB phase shift function in a power series in the strength of the potential in terms of the impact parameters b:

$$\chi(b)=\sum_{n=0}^{\infty}\chi^n(b), \tag{24}$$

where

$$\chi^n(b)=-\frac{2k[m/(\hbar k)^2]^{n+1}}{(n+1)!\,b^{2n}}\left[b^2\left(1+b\frac{d}{db}\right)\right]^n \times\int_0^\infty U_{op}^{n+1}(r)\,dz \tag{25}$$

with $r=(b^2+z^2)^{1/2}$. The zeroth-order eikonal phase shift and its first-order correction are given by

$$\chi^0(b)=-\frac{2m}{\hbar^2 k}\int_0^\infty U_{op}(r)\,dz, \tag{26}$$

$$\chi^1(b)=-\frac{m^2}{\hbar^4 k^3}\left(1+b\frac{d}{db}\right)\times\int_0^\infty U_{op}^2(r)\,dz. \tag{27}$$

The first-order eikonal correction term of the phase shift, $\chi^1(b)$ in Eq. (27), can further be expressed as following

$$\chi^1(b)=-\frac{2m^2}{\hbar^4 k^3}\int_0^\infty\left[U_{op}^2(r)+rU_{op}(r)\frac{dU_{op}(r)}{dr}\right]dz. \tag{28}$$

Restricting ourselves to the first-order correction term, the closed expression of the phase shift function may be written as

$$\chi(b)=-\frac{2m}{\hbar^2 k}\int_0^\infty \tilde{U}_{op}(r)\,dz, \tag{29}$$

where

$$\tilde{U}_{op}(r)=U_{op}(r)\left\{1+\frac{m}{\hbar^2k^2}\left[U_{op}(r)+r\frac{dU_{op}(r)}{dr}\right]\right\}. \tag{30}$$

The phase shift $\chi(b)$ in Eq. (29) is Abel's type [6, 8] and the inverse solution of this equation has the form

$$\widetilde{U}_{op}(r) = \frac{2E}{k\pi}\frac{1}{r}\frac{d}{dr}\int_r^\infty \frac{\chi(b)}{\sqrt{b^2-r^2}}\, b\, db. \quad (31)$$

The optical potential obtained from the McIntyre parametrization is now $\widetilde{U}_{op}(r)$ which includes the first-order eikonal correction term instead of $U_{op}(r)$. The first-order corrected optical potential $\widetilde{U}_{op}(r)$ is related to $U_{op}(r)$ through Eq. (30). By using the optical potential $\widetilde{U}_{op}(r)$ calculated from solving the inversion scattering problem, the optical potential $U_{op}(r)$ may be obtained from Eq. (30). However, this presents an involved computional problem, which can be solved by some iteration methods. However, at high energies which concerns us, the first-order eikonal correction is expected to be small, hence it is a reasonably good approximation to replace $U_{op}(r)$ in the correction factor in Eq. (30) by $\widetilde{U}_{op}(r)$ as reported in Ref. [8]. Thus we have

$$U_{op}(r) \approx \frac{\widetilde{U}_{op}(r)}{1+\frac{m}{\hbar^2k^2}[\widetilde{U}_{op}(r)+r\frac{d\widetilde{U}_{op}(r)}{dr}]}. \quad (32)$$

Since $\widetilde{U}_{op}(r) = \widetilde{V}(r)+i\widetilde{W}(r)$, the optical potential $U_{op}(r) = V(r)+iW(r)$ in the above equation (32) can be expressed in the form

$$V(r) = \frac{\widetilde{V}(r)[1+\frac{m}{\hbar^2k^2}(\widetilde{V}(r)+r\frac{d\widetilde{V}(r)}{dr})]+\widetilde{W}(r)[\frac{m}{\hbar^2k^2}(\widetilde{W}(r)+r\frac{d\widetilde{W}(r)}{dr})]}{[1+\frac{m}{\hbar^2k^2}(\widetilde{V}(r)+r\frac{d\widetilde{V}(r)}{dr})]^2+[\frac{m}{\hbar^2k^2}(\widetilde{W}(r)+r\frac{d\widetilde{W}(r)}{dr})]^2} \quad (33)$$

and

$$W(r) = \frac{\widetilde{W}(r)[1+\frac{m}{\hbar^2k^2}(\widetilde{V}(r)+r\frac{d\widetilde{V}(r)}{dr})]-\widetilde{V}(r)[\frac{m}{\hbar^2k^2}(\widetilde{W}(r)+r\frac{d\widetilde{W}(r)}{dr})]}{[1+\frac{m}{\hbar^2k^2}(\widetilde{V}(r)+r\frac{d\widetilde{V}(r)}{dr})]^2+[\frac{m}{\hbar^2k^2}(\widetilde{W}(r)+r\frac{d\widetilde{W}(r)}{dr})]^2}. \quad (34)$$

Using Eqs. (33) and (34), we can estimate the effect of first-order eikonal correction on the calculated optical potential by inverse method.

TABLE I: Parameters in the McIntyre strong absorption model for ^{12}C + ^{12}C elastic scattering at E_{lab} = 2400 MeV.

$l_{g'}$	$\Delta_{g'}$	l_g	Δ_g	μ
80.11	14.49	73.89	13.86	0.5686

IV. RESULTS AND CONCLUSIONS

We have calculated the optical potential by inversion for ^{12}C + ^{12}C elastic scattering at E_{lab} = 2400 MeV by using the zeroth-order eikonal approximation and its first-order corrections. In order to calculate the optical potential by inversion, five McIntyre parameters μ, b_0, b_0',d and d' are required. It is found[3] that the McIntyre SAM gave reasonable fit to the differential cross section data of this system. The differential cross sections calculated by Mermaz *et al.* [3] are shown in figure 1 and the pertinent values of the parameters as determined are given in table I for completeness.

Using these McIntyre phase shift parameters, impact parameters b_0, b_0', and diffusivity quantities d , d' were obtained. Note that Coulomb effects have been substracted off in evaluating the parameters cited above. Using these parameters and solving two sets of nonlinear simultaneous Eqs.(16) -(17) and Eqs. (18)-(19) with the iteration procedure such as the Newton method[14], we can obtain numerical values of R, Δ, R' and Δ'. Their values are R=2.931 fm, Δ =0.754 fm, R'=4.929 fm and Δ'=0.711 fm. From Eqs. (21) and (22), the parameters α=3.150 fm and β=0.349 fm have been also calculated. Accordingly, we obtained $\widetilde{V}_0$=14.9 MeV and $\widetilde{W}_0$=117.9 MeV.

Figure 2 shows the real and imaginary parts of optical potential calculated from the inversion procedure based on the McIntyre parametrization of S -matrix. In this figure, the solid and dashed curves are the optical potentials with and without the first-order eikonal correction, respectively. And the dotted curves are the results from the optical model analysis given in Ref. [13]. For the real potential, the first-order eikonal correction is relatively large , where maximum reduction is about 37 % at small r region as shown in figure 3. But it is significant in the surface region around the strong absorption radius. The large discrepancies around $r = 0$ is of little significant, since in this region the condition $\frac{|U_{op}(r)|}{E} \ll 1$ which forms the basis of the high-energy expansion of the phase function may not be well satisfied and hence the potentials calculated in this region by the eikonal inverse method may not be as reliable as in the surface region where this condition is satisfied well. While in the case of imaginary potential, it can be seen that the first-order eikonal correction is negligibly samll as shown in Fig. 2(b) and Fig.3, where maximum reduction is about 2 % at small r region. As a whole, the potential obtained in the present study agrees with one from the optical model analysis [13] in the surface region around the strong absorption radius. We can see that in the surface region, the first-order eikonal correction brings the calculated real potential closer to the phenomeological one by optical model analysis[13]. Since the elastic scattering data are sensitive mainly to the surface region, somewhat big discrepancies at samll r between optical potentials from the inverse method and optical model analysis [13] are of little significant.

In this paper, we used the McIntyre phase shift parameters obtained from Mermaz *et al.*[3] to obtain the heavy-ion optical potential by inverse method. Using the relation between the optical potential and McIntyre phase shift, we calculated the ^{12}C + ^{12}C

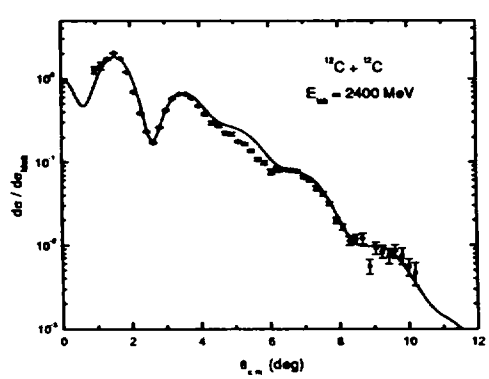

FIG. 1: Elastic scattering angular distributions for ^{12}C + ^{12}C system at E_{lab} = 2400 MeV. The solid circles denote the observed data taken from Hostachy *et al.*[13]. The solid curve is the calculated results from the Mermaz *et al.*[3].

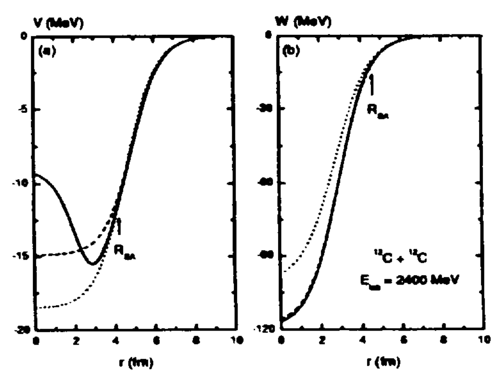

FIG. 2: (a) Real $V(r)$ and (b) imaginary $W(r)$ parts of ^{12}C + ^{12}C optical potential at E_{lab} = 2400 MeV. The solid and dashed curves denote the calculated optical potentials with and without the first-order eikonal correction, respectively, while the dotted curves are the results from the optical model analysis[13] .

Woods-Saxon type optical potential by solving the inverse scattering problem. As a whole, the optical potential obtained in this study agrees with the phenomenological one obtained from optical model analysis in the surface region around the strong absorption radius. We found that the agreement of real potential obtained from inverse method with the result of optical model analysis is fairly good compared to one of the imaginary potential. The effect of first-order eikonal correction on the determination of optical potential by inversion was investigated for this system. It is seen in the surface region around the strong absorption radius that the real potential including the first-order eikonal correction improves the agreement with the Woods-Saxon type one by the optical model analysis compared to the result of the zeroth-order eikonal approximation for this system. We can see that first-order eikonal correction is significant in the surface region around the strong absorption radius for the real potential, while it is negligibly small for imaginary potential.

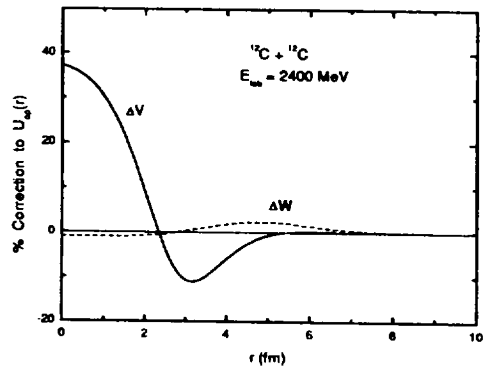

FIG. 3: First-order correction (in %) to the calculated real and imaginary parts of ^{12}C + ^{12}C optical potential at E_{lab} = 2400 MeV. The solid and dashed curves present the corrections to the real and imaginary parts of the potential, respectively.

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E_{lab} = 2400 MeV인 ^{12}C + ^{12}C 탄성산란에서 역방법에 의한 광학퍼텐셜의 제1차 Eikonal 보정

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McIntyre 위상이동과 Woods-Saxon형 광학퍼텐셜 사이의 관계를 이용한 역산란 문제를 풀어서 E_{lab} = 2400 MeV에서의 ^{12}C + ^{12}C 탄성산란에 대한 광학퍼텐셜을 계산하였다. 중이온 광학퍼텐셜에 대한 제1차 Eikonal 보정 효과가 조사되었다. 실수퍼텐셜에 대한 제1차 Eikonal 보정효과는 이 산란계의 표면영역 주위에서 중요함을 알 수 있었다.