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# Coulomb-Modified Eikonal Phase Shift Analysis for $^{16}O + ^{28}Si$ Elastic Scattering at $E_{lab} = 1503$ MeV in the Approximated Hyperbolic Trajectory

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We presented the Coulomb-modified eikonal phase shift based on the approximated hyperbolic trajectory for heavy-ion elastic scatterings. It has been applied successfully to elastic scatterings of $^{16}O + ^{28}Si$ system at $E_{lab} = 1503$ MeV. The presence of a nuclear rainbow is evidenced from classical deflection function. The Fraunhöfer oscillations observed in the elastic angular distributions could be explained due to the interference between the near- and far-side amplitudes.

## I. INTROCUCTION

The study of elastic scattering is a basic ingredient to understand more complicated reaction processes in heavy-ion collision. To understand and systematize measured data of heavy-ion elastic scattering, various methods have been used. The interpretation and description of elastic scattering phenomena in heavy ion reactions has been greatly facilitated through the application of the semiclassical method. The semiclassical method is a useful approximation technique when the de Broglie wavelength of the incident particle is sufficiently short compared with the distance in which the potential varies appreciably. The widely used method for analysis of the elastic scattering data is the WKB approximation [1,2]. The eikonal phase shift is derived from the the integral equation by further approximating the WKB results.

The eikonal approximation has been found to be a very effective approach to describe the heavy-ion elastic scattering. There has been a great deal of effort [3-7] in describing elastic scattering processes between heavy-ions within the framework of the eikonal approximation methods. The first- and third-order non-eikonal corrections to the Glauber model have been developed to know the possibility of observing a bright interior in the nucleus "viewed" by intermediate energy alpha particles ($E_\alpha$=172.5 MeV), as a probe for the $^{58}Ni$ nucleus [6]. The use of the distance of closest approach as an effective impact parameter in the eikonal formula provided a simple and efficient way for studying heavy-ion scattering at relatively low energies [7]. In a recent paper [8], we have presented the first- and second-order corrections to the zeroth-order eikonal phase shifts for heavy-ion elastic scatterings based on Coulomb trajectories of colliding nuclei and it has been applied satisfactorily to the $^{16}O + ^{40}Ca$ and $^{16}O + ^{90}Zr$ systems at $E_{lab}$=1503 MeV.

The Glauber model approach based on the hyperbolic trajectory for the description of the heavy-ion reaction cross section has been presented to extend to lower energies [9]. Gupta *et al.* [10] described the modifications in the trajectory due to the additional nuclear field along with its noneikonal nature and these modification improved the description of the reaction cross section in the framework of the Glauber model. The Coulomb-modified eikonal model formalism based on hyperbolic trajectory for the description of heavy-ion elastic scattering was described and it has been applied satisfactorily to elastic scatterings of $^{12}C + ^{12}C$ system at $E_{lab}$=240, 360 and 1016 MeV [11].

In this paper, we present the Coulomb-modified eikonal phase shift based on the approximated hyperbolic trajectory and apply it the elastic scattering angular distributions of $^{16}O + ^{28}Si$ system at $E_{lab}$ = 1503 MeV. In section II, we describe the Coulomb-modified eikonal phase shift formalism based on the approximated hyperbolic trajectory. Sec. III contains the results and discussions for the Coulomb-modified eikonal phase shift analysis. Finally, concluding remarks are presented in section IV.

## II. FORMALISM

If there is a single turning point in the radial Schrödinger equation, a first-order WKB expression for the nuclear elastic phase shifts $\delta_L$, taking into account the deflection effect due to Coulomb field, can be written as [12,13]

$$\delta_L = \int_{r_t}^{\infty} k_L(r)\,dr \;-\; \int_{r_c}^{\infty} k_c(r)\,dr, \quad (1)$$

where $r_t$ and $r_c$ are the turning points corresponding to the local wave numbers $k_L(r)$ and $k_c(r)$ given by

$$k_L(r) = k\Big[1 - \Big(\frac{2\eta}{kr} + \frac{(L+\frac{1}{2})^2}{k^2r^2} + \frac{U_N(r)}{E}\Big)\Big]^{1/2}, \quad (2)$$

$$k_c(r) = k\Big[1 - \Big(\frac{2\eta}{kr} + \frac{(L+\frac{1}{2})^2}{k^2r^2}\Big)\Big]^{1/2}, \quad (3)$$

where $\eta$ is the Sommerfeld parameter, and $U_N(r)$ the nuclear potential. If we consider the nuclear potential as a perturbation, the nuclear phase shift can be written as [14]

$$\delta_L(r_c) = -\frac{\mu}{\hbar^2 k}\int_{r_c}^{\infty} \frac{rU_N(r)}{\sqrt{r^2 - r_c^2}}dr, \quad (4)$$

where $r_c$ is the Coulomb distance of closest approach given by

$$r_c = \frac{1}{k}\left\{\eta + \left[\eta^2 + (L+\frac{1}{2})^2\right]^{1/2}\right\}, \quad (5)$$

and $r$ is given in the cylindrical coordinate system as

$$r^2 = y^2 + z^2. \quad (6)$$

In the heavy-ion scattering, the orbit describing a projectile may be considered as one branch of a hyperbola. So, we incorporate the deviation in the eikonal trajectory due to the Coulomb field by replacing the impact parameter $b$ by the coordinate $y$ so that instead of $y^2 = \text{const} = b^2$ we have $y^2$ given by

$$y^2 = r_c^2 + Cz^2, \quad (7)$$

where $C$ is a constant independent of $z$.

The hyperbolic trajectory with respect to the center of the potential in the case of the Coulomb potential is given by [10]

$$\frac{k^2(y-s)^2}{\eta^2} - \frac{k^2z^2}{L^2} = 1, \quad (8)$$

where

$$s = (\eta^2 + L^2)^{1/2}/k. \quad (9)$$

If Eq.(8) is arranged, the following expression is obtained

$$y^2 = r_c^2 + \frac{2c\eta}{k}\Big(\sqrt{1+\frac{k^2}{L^2}z^2} - 1\Big) + \frac{\eta^2}{L^2}z^2. \quad (10)$$

Now by assuming $kz/L < 1$, we get

$$y^2 \simeq r_c^2 + \frac{\eta k}{L^2}r_c z^2. \quad (11)$$

By comparing Eq.(7) with Eq.(11), the quantity $C$ can be obtained as

$$C = \frac{\eta k}{L^2}r_c. \quad (12)$$

Using Eq.(4), Eq.(6) and (7), the nuclear phase shift $\delta_L(r_c)$ can further be expressed as following

$$\delta_L(r_c) = -\frac{\mu}{\hbar^2 k}(C+1)^{1/2}\int_0^{\infty} U_N(r)dz, \quad (13)$$

with $r = \sqrt{r_c^2 + (C+1)z^2}$.

By taking $U_N(r)$ as the optical Woods-Saxon potential given by

$$U_N(r) = -\frac{V_0}{1+e^{(r-R_v)/a_v}} - i\frac{W_0}{1+e^{(r-R_w)/a_w}}, \quad (14)$$

with $R_{v,w} = r_{v,w}(A_1^{1/3} + A_2^{1/3})$, we can use the phase shift in the general expression for the elastic scattering amplitude. The elastic scattering cross sections is then obtained from the scattering amplitude

$$f(\theta) = f_R(\theta) + \frac{1}{ik}\sum_{L=0}^{\infty}(L+\frac{1}{2})\exp(2i\sigma_L) \times (S_L^N - 1)P_L(\cos\theta), \quad (15)$$

where $f_R(\theta)$ is the usual Rutherford scattering amplitude and $\sigma_L$ the Coulomb phase

TABLE I: Parameters of the fitted Woods-Saxon potential for $^{16}O$ + $^{28}Si$ system at $E_{lab}$ = 1503 MeV. Values are taken from Ref. [16]

| $V_o$ (MeV) | $r_v$ (fm) | $a_v$ (fm) | $W_o$ (MeV) | $r_w$ (fm) | $a_w$ (fm) |
|---|---|---|---|---|---|
| 100 | 0.892 | 0.905 | 50.5 | 0.992 | 0.780 |

TABLE II: Nuclear rainbow angle ( $\theta_{n.r.}$), strong absorption distance ($R_{1/2}$), reaction cross sections ($\sigma_R$) from the Coulomb-modified eikonal phase shift analysis based on the approximated hyperbolic trajectory, for $^{16}O$ + $^{28}Si$ system at $E_{lab}$ = 1503 MeV.

| $\theta_{n.r.}$ | $R_{1/2}$ (fm) | $\sigma_R'(mb)$ | $\sigma_R(mb)$ |
|---|---|---|---|
| -4.7° | 7.20 | 1630 | 1722 |

shift. The nuclear $S$-matrix elements $S_L^N$ in this equation can be expressed by the nuclear phase shift $\delta_L$ as

$$S_L^N = e^{2i\delta_L}. \quad (16)$$

A 'sharp cut-off model' proposed by Blair [15] reproduces some of the main features of the angular distribution. In this model, Eq. (16) can be replaced

$$S_L^N = \begin{cases} 0, & L \le L_g \\ \exp(2i\delta_L), & L > L_g \end{cases} \quad (17)$$

where $L_g$ is a grazing angular momentum.

## III. RESULTS AND DISCUSSIONS

As in the preceding section, we have calculated the elastic differential cross sections for $^{16}O$ + $^{28}Si$ system at $E_{lab}$ = 1503 MeV by using the Coulomb-modified eikonal phase shift based on the approximated hyperbolic trajectory. The measured angular distributions [16] for $^{16}O$ + $^{28}Si$ elastic scatterings at $E_{lab}$= 1503 MeV are given as the ratio of the experimental cross section to the Rutherford cross section. These data exhibit a Fraunhöfer diffraction pattern at small scattering angles followed by an exponential fall-off at large angles. Table 1 show the parameters of the Woods-Saxon potential taken from Roussel-Chomz *et al.*[16]. The calculated results of the differential cross sections for the elastic scattering of $^{16}O$ + $^{28}Si$ system at $E_{lab}$= 1503 MeV are depicted in Fig. 1 together with the observed data [16]. In Fig.1, the dashed curve is the sharp cut-off calculation with $L_g$ = 97, while the solid curve is the full calculation. Two calculated angular distributions are nearly identical at forward angles but are qualitatively different at large angles. The Coulomb-modified eikonal phase shift based on the approximated hyperbolic trajectory provides reasonable fit to the refractive structure of the cross section at large angles.

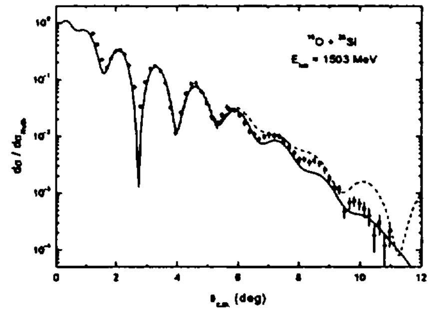

FIG. 1: Elastic scattering angular distributions for $^{16}O$ + $^{28}Si$ system at $E_{lab}$ = 1503 MeV. The solid circles denote the observed data taken from Roussel-Chomz *et al.* [16]. The solid curve is the calculated results based on the approximated hyperbolic trajectory. The dashed curve is sharp cut-off calculation with $L_g$ = 97.

In Fig. 2, we plot the transmission function, $T_L = 1 - |S_L^N|^2$, in the approximated hyperbolic trajectory as a function of angular momentum $L$, along with the deflection function. We found in figure 2(a) that lower partial waves are totally absorbed and $T_L$ decrease rapidly in the narrow localized angular momentum zone. $T_L$ distribution can be used to define the strong absorption distance $R_{1/2}$, a quantity which characterizes the system with respect to strong absorption. The $R_{1/2}$ value in table II is the distance of closest approach based on the approximated hyperbolic trajectory associated with $T_{L_{1/2}} = \frac{1}{2}$. For internuclear distances smaller than $R_{1/2}$ the ab-

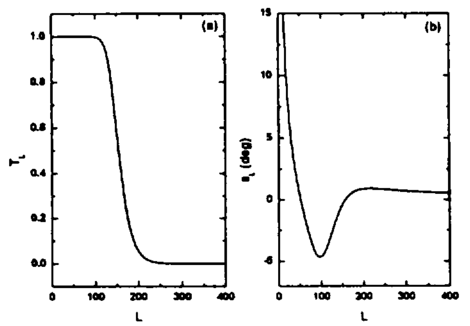


FIG. 2: (a) Transmission function $T_L$ and (b) deflection function $\theta_L$ from the Coulomb-modified eikonal model based on the approximated hyperbolic trajectory for $^{16}O$ + $^{28}Si$ system at $E_{lab}$ = 1503 MeV.

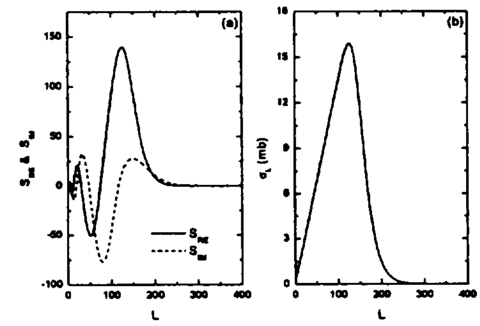


FIG. 3: (a) Real ($S_{RE}$) (solid curve) and imaginary ($S_{IM}$) (dashed curve) parts of $\frac{1}{i}(L + \frac{1}{2})\exp(2i\sigma_L)(S_L^N - 1)$, and (b) the partial reaction cross section $\sigma_L$.

sorption dominates, whereas for values larger than $R_{1/2}$ partial waves are mostly deflected in the elastic channel. We can see in table II that the strong absorption distance provides a good measurement of reaction cross section in terms of $\sigma'_R = \pi R_{1/2}^2$. The presence of nuclear rainbow can be proved by investigating deflection function given by the formula $\theta_L = 2d(\sigma_L + \mathrm{Re}\delta_L)/dL$. The term "nuclear rainbow" shows the differential cross section for scattering to the negative angle from the far-side of the target. The deflection function in Fig.2(b) shows that the nuclear rainbow angle value is $\theta_{nr} = -4.7^{\circ}$ for $^{16}O$ + $^{28}Si$ system at $E_{lab}$= 1503 MeV. Such a negative angle proves a presence of the nuclear rainbow.

Figure 2(b) shows that the angular momentum in the nuclear rainbow region is $L_g \sim 97$, and the corresponding value of modulus of $S$-matrix $|S_L^N|$ at $L_g$ is 0.038 for $^{16}O$ + $^{28}Si$ system at $E_{lab}$= 1503 MeV. Nevertheless, this small $|S_L^N|$ value contributes to the fall-off pattern of the cross section at large angles as shown in Fig.1. The contributions of the partial waves with $L < L_g$ to the cross section can be estimated from sharp cut-off calculation setting the $S$-matrix to zero for all $L$-values lower than the chosen $L_g$-value. In other words, the partial wave summations for the cross sections is performed from $L_g$ to $L = \infty$. The sharp cut-off calculation is shown in Fig.1 as dashed curve. The cross section at forward angles is not changed by these sharp cut-off calculation, but at large angles there are some differences between solid and dashed curves. The differences are related to the contributions from $L < L_g$. We can see that a genuine refraction effect is arising from the region of the rainbow minimum.

In figure 3, we plot the $\frac{1}{i}(L + \frac{1}{2})\exp(2i\sigma_L)(S_L^N - 1)$ along with the partial reaction cross section. The real($S_{RE}$) and imaginary( $S_{IM}$) parts of $\frac{1}{i}(L + \frac{1}{2})\exp(2i\sigma_L)(S_L^N - 1)$ represent the partial wave contributions to the cross sections in terms of orbital angular momentum $L$. The partial wave contributions, $\sigma_L = \frac{\pi}{k^2}(2L + 1)(1 - |S_L^N|^2)$, to the total reaction cross section as a function of $L$ are also presented in Fig. 3(b). We found in this figure that the value of the partial wave reaction increases linearly up to $L$ = 126. Beyond this $L$ value, the partial reaction cross section decreases quardratically, indicating that the regions of higher partial waves almost not contribute to the total reaction cross section.

In order to understand the nature of angular distributions for the $^{16}O$ + $^{28}Si$ system at $E_{lab}$ = 1503 MeV, the near- and far-side decompositions of scattering amplitudes are also performed with Coulomb-modified eikonal phase shift based on the approximated hyperbolic trajectory by following the

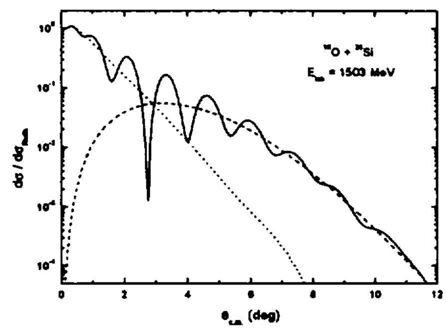

FIG. 4: Differential cross section (solid curve), near-side contribution (dotted curve), and far-side contribution (dashed curve) following the Fuller's formalism [21] from the Coulomb-modified eikonal model based on the approximated hyperbolic trajectory.

Fuller's formalism [17]. The near-side amplitude represents contributions from waves deflected to the direction of $\theta$ on the near-side of the scattering center and the far-side amplitude represents contributions from waves traveling from the opposite, far side of the scattering center to the same angle $\theta$. The contributions of the near- and far-side components to the elastic cross sections for $^{16}O$ + $^{28}Si$ system are shown in figure 4 along with the total differential cross section. The total differential cross section is not just a sum of the near- and far-side components but contains interference between two amplitudes as seen in this figure. The Fraunhöfer oscillations observed on the elastic scattering cross section of $^{16}O$ + $^{28}Si$ system are due to the interference between the near- and far-side components. The magnitudes of the near- and far-side contributions are about the same around 2.9° for this system. The interference oscillations between the near- and far-side cross sections have maximum amplitude near this angle due to the enhanced far-side ones. The near-side contribution dominates for small angles and the far-side one for large angles.

## IV. CONCLUDING REMARKS

In this paper, we have presented the Coulomb-modified eikonal phase shift based on the approximated hyperbolic trajectory. It has been applied satisfactorily to the elastic scatterings of $^{16}O$ + $^{28}Si$ system at $E_{lab}$= 1503 MeV. The calculated result leads to a reasonable agreement with experimental data in this system. The cross sections at forward angles are not changed by sharp cut-off calculations with the grazing angular momentum $L_g$ related with nuclear rainbow region, but at large angles there are some differences between full and sharp cut-off calculations. The differences are related to the contributions from $L < L_g$ to the cross sections. The strong absorption distance provides a good measurement of reaction cross section in terms of $\sigma_R^{'} = \pi R_{1/2}^2$. The value of the partial wave reaction cross section increases linearly up to $L = 126$. Beyond this $L$ value, the partial reaction cross section decreases quardratically, indicating that the regions of higher partial waves almost not contribute to the total reaction cross section. The presence of nuclear rainbow is evidenced by the classical deflection function. The sharp cut-off calculation demonstrates that a genuine refraction effect is arising from the region of the rainbow minimum. Through near- and far-side decompositions of the elastic cross section for $^{16}O$ + $^{28}Si$ system at $E_{lab}$= 1503 MeV, the Fraunhöfer oscillations at intermediate angles are due to the interference between the near- and far-side amplitudes. The elastic scattering pattern at large angles was dominated by the refraction of the far-side trajectories.

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# 근사된 쌍곡선 궤적에서 입사에너지가 1503 MeV 인 $^{16}O$ + $^{28}Si$ 탄성산란에 대한 쿨롱-수정된 Eikonal 위상이동 분석

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## 요 약

근사된 쌍곡선 궤적에 기초하여 중이온 탄성산란에 대한 쿨롱-수정된 Eikonal 위상이동 분석방법을 나타내었다. 이러한 분석방법은 입사에너지가 1503 MeV 인 $^{16}O$ + $^{28}Si$ 계의 탄성산란에 성공적으로 적용될 수 있었다. 고전적 편향함수를 통하여 핵 무지개 각의 존재를 확인 할 수 있었다. 탄성산란 각분포에서 관측된 Fraunhofer 진동현상은 근측과 원측 진폭들 사이의 간섭현상으로 설명될 수 있었다.