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INITIAL CONTROL FOR AN ADSORBATE-INDUCED PHASE TRANSITION MODEL

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ABSTRACT. In this paper we are concerned with the initial control problem for an adsorbate-induced phase transition model. That is, we show the existence of the initial control and derive the optimality conditions by showing the differentiability of the cost functional.

1. INTRODUCTION

We consider the following initial control problem

$$\text{(P)} \qquad \text{minimize} \;\; J(u,v)$$

with the cost functional $J(u,v)$ of the form

$$J(u,v) = \int_0^T \|y(u,v)-y_d\|^2_{H^3(\Omega)}dt + \int_0^T \|\rho(u,v)-\rho_d\|^2_{H^2(\Omega)}dt + \gamma\{\|u\|^2_{H^3(\Omega)} + \|v\|^2_{H^2(\Omega)}\}, \qquad (u,v) \in H^3(\Omega) \times H^2(\Omega)$$

where $y = y(u,v)$ and $\rho(u,v)$ is governed by the adsorbate-induced phase transition model:

$$\begin{aligned}
&\frac{\partial y}{\partial t} = a\Delta y - dy(y+\rho-1)(1-y) && \text{in } \Omega \times (0,T],\\
&\frac{\partial \rho}{\partial t} = b\Delta\rho + c\nabla\cdot\{\rho(1-\rho)\nabla\chi(y)\} - fe^{\alpha\chi(y)}\rho \\
&\qquad\qquad -g\rho + h(1-\rho) && \text{in } \Omega \times (0,T],\\
&\frac{\partial y}{\partial n} = \frac{\partial \rho}{\partial n} = 0 && \text{on } \partial\Omega \times (0,T],\\
&y(x,0) = u(x), \quad \rho(x,0) = v(x) && \text{in } \Omega.
\end{aligned} \tag{1.1}$$

Here, Ω is a bounded region in $\mathbb{R}^2$ of C^3 class. $n = n(x)$ is the outer normal vector at a boundary point $x \in \partial\Omega$ and $\frac{\partial}{\partial n}$ denotes the differentiation along the vector n. $y(x,t)$ denotes the order parameter which represents the structural state of the surface at a position $x \in \Omega$ and a time $t \in [0,\infty)$, and $\rho(x,t)$ the adsorbate coverage of the surface Ω by a specific kind of molecules. $dy(y+\rho-1)(1-y)$ shows that the surface has two stable states. $c\nabla\cdot\{\rho(1-\rho)\nabla\chi(y)\}$ shows the advection of ρ over Ω induced by the gradient of the local chemical potential $\chi(y)$ for with mobility $1-\rho$. $fe^{\chi(y)}$ denotes the desorption rate of the molecules depending on $\chi(y)$. $\chi(y)$ is assumed to be given smooth function for y, prototype of $\chi(y)$ is

$$\chi(y) = -y^2(3 - 2y).$$

g denotes the desorption rate of the molecules by a chemical reaction. h denotes the adsorption rate determined by the pressure of gaseous molecules and the fraction of the surface being adsorbate-free, $(1 - \rho)$. a and b are positive diffusion constants. c, d, f, g, h, α and γ are assumed to be positive constants.

The adsorbate-induced phase transition model was introduced by Hildebrand et al. [4] and Mikhailov et al. [6]. They showed that microreactors with submicroreactor and nanometer sizes may spontaneously develop in surface chemical reactions by a nonequilibrium self-organization process. The self-organized micrometers represent localized structure resulting from the interplay between the reaction, diffusion, and an adsorbate-induced structure transformation of the surface. They assumed also that the free energy is associated with the first-order surface phase transition due to the adsorption of the chemical substance.

In a practical situations, the initial data of the state is unknown or only known partially. This kind of problem is hard to modelized and it seems that there is a wide field not very much explored yet. The way used in this paper is the adjustment of the initial data in order to obtain the desired state from the observed data. Such problem is treated as an optimal control problem with the initial data serving as the control.

Many papers have already been published to study the control problems for nonlinear parabolic equations([1], [2], [5], [8]). The method used in this paper is very analogous to that in [8] handling the chemotaxis-diffusion equations. The advection terms in the chemotaxis models are given in the form $\nabla \cdot \{\rho \nabla \chi(y)\}$. Therefore, the present advection term $\nabla \cdot \{\rho(1-\rho)\nabla\chi(y)\}$ in (1.1) has stronger nonlinearity in ρ, which reflects some technical difficulty.

The paper is organized as follows. In Section 2, we recall some known results and show the existence of the optimal control. Section 3 is devoted to obtaining the optimality conditions for the optimal control.

Notations. $\mathbf{R}$ denotes the sets of real numbers. Let I be an interval in $\mathbf{R}$. $L^p(I; \mathcal{H})$, $1 \le p \le \infty$, denotes the L^p space of measurable functions in I with values in a Hilbert space $\mathcal{H}$. $C(I; \mathcal{H})$ denotes the space of continuous functions in I with values in $\mathcal{H}$. For simplicity, we shall use a universal constant C to denote various constants which are determined in each occurrence in a specific way by δ, M, and so forth. In a case when C depends also on some parameter, say θ, it will be denoted by C_θ.

2. Mathematical setting

Let $A_1 = -a\Delta + a$ and $A_2 = -d\Delta + g$ with the same domain $\mathcal{D}(A_i) = H^2_n(\Omega) = \{z \in H^2(\Omega); \frac{\partial z}{\partial n} = 0 \text{ on } \partial\Omega\}$ $(i = 1, 2)$. Then, A_i are two positive definite self-adjoint operators in $L^2(\Omega)$. $\mathcal{D}(A_i^\theta) = H^{2\theta}(\Omega)$ for $0 \le \theta < \frac{3}{4}$, and $\mathcal{D}(A_i^\theta) = H^{2\theta}_n(\Omega)$ for $\frac{3}{4} < \theta \le \frac{3}{2}$ (see [10]). We set two product Hilbert spaces $\mathcal{V} \subset \mathcal{H}$ as

$$\mathcal{V} = H_n^3(\Omega) \times H_n^2(\Omega), \quad \mathcal{H} = H_n^2(\Omega) \times H^1(\Omega).$$

By identifying $\mathcal{H}$ with its dual space, we consider $\mathcal{V} \subset \mathcal{H} = \mathcal{H}' \subset \mathcal{V}'$. It is then seen that

$$\mathcal{V}' = H^1(\Omega) \times L^2(\Omega),$$

with the duality product

$$\langle \Phi, Y \rangle_{\mathcal{V}' \times \mathcal{V}} = \langle A_1^{1/2}\zeta, A_1^{3/2}y \rangle_{L^2} + (\varphi, A_2\rho)_{L^2}, \quad \Phi = \begin{pmatrix} \zeta \\ \varphi \end{pmatrix}, \; Y = \begin{pmatrix} y \\ \rho \end{pmatrix}.$$

We denote the norms of $\mathcal{V}$, $\mathcal{H}$, and $\mathcal{V}'$ by $\|\cdot\|$, $|\cdot|$, and $\|\cdot\|_*$, respectively. $(\cdot,\cdot)$ and $\langle\cdot,\cdot\rangle$ denote the scalar product of $\mathcal{H}$ and the pairing between $\mathcal{V}$ and $\mathcal{V}'$.

We set also a symmetric bilinear form on $\mathcal{V} \times \mathcal{V}$:

$$a(Y, \tilde{Y}) = (A_1 y, A_1 \tilde{y})_{L^2} + (A_2^{1/2}\rho, A_2^{1/2}\tilde{\rho})_{L^2}, \qquad Y = \begin{pmatrix} y \\ \rho \end{pmatrix}, \tilde{Y} = \begin{pmatrix} \tilde{y} \\ \tilde{\rho} \end{pmatrix} \in \mathcal{V}.$$

Obviously, the form satisfies

$$|a(Y, \tilde{Y})| \le M\|Y\|\|\tilde{Y}\|, \quad Y, \tilde{Y} \in \mathcal{V},$$
$$a(Y, Y) \ge \delta\|Y\|^2, \quad Y \in \mathcal{V}$$

with some δ and $M > 0$. This form then defines a linear isomorphism $A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}$ from $\mathcal{V}$ to $\mathcal{V}'$, and the part of A in $\mathcal{H}$ is a positive definite self-adjoint operator in $\mathcal{H}$ with the domain $\mathcal{D}(A) = H_n^4(\Omega) \times H_n^3(\Omega)$.

(1.1) is, then, formulated as an abstract equation

$$\begin{aligned} &\frac{dY}{dt} + AY = F(Y), \quad 0 < t \le T, \\ &Y(0) = U \end{aligned} \tag{2.1}$$

in the space $\mathcal{V}'$. Here, $F(\cdot) : \mathcal{V} \to \mathcal{V}'$ is the mapping

$$F(Y) = \begin{pmatrix} ay + dy(y + \rho - 1)(1 - y) \\ c\nabla \cdot \{\rho(1-\rho)\nabla\chi(y)\} - fe^{\alpha\chi(y)}\rho + h(1 - \rho) \end{pmatrix}, \qquad Y = \begin{pmatrix} y \\ \rho \end{pmatrix}.$$

Here, U is defined by $U = \binom{u}{v}$.

As verified in [9, Sec. 3], $F(\cdot)$ satisfies the following conditions:

(f.i) For each $\eta > 0$, there exists an increasing continuous function $\phi_\eta : [0, \infty) \to [0, \infty)$ such that

$$\|F(Y)\|_* \le \eta\|Y\| + \phi_\eta(|Y|), \quad Y \in \mathcal{V},$$
$$|F(Y)| \le \eta\|Y\|_{\mathcal{D}(A)} + \phi_\eta(\|Y\|), \quad Y \in \mathcal{D}(A).$$

(f.ii) For each $\eta > 0$, there exists an increasing continuous function $\psi_\eta : [0, \infty) \to [0, \infty)$ such that

$$\|F(\tilde{Y})-F(Y)\|_* \le \eta\|\tilde{Y}-Y\| + (\|\tilde{Y}\|+\|Y\|+1)\psi_\eta(|\tilde{Y}|+|Y|)|\tilde{Y}-Y|, \quad \tilde{Y},Y\in\mathcal{V},$$
$$|F(\tilde{Y})-F(Y)| \le \eta\|\tilde{Y}-Y\|_{\mathcal{D}(A)} + (\|\tilde{Y}\|_{\mathcal{D}(A)}+\|Y\|_{\mathcal{D}(A)}+1)\psi_\eta(\|\tilde{Y}\|+\|Y\|)\|\tilde{Y}-Y\|, \quad \tilde{Y},Y\in\mathcal{D}(A).$$

We then obtain the following result (For the proof, see Ryu and Yagi [8]).

Theorem 2.1. *Let* (a.i), (a.ii), (f.i), *and* (f.ii) *be satisfied. Then, for any* $U\in\mathcal{V}$, *there exists a unique weak solution*

$$Y\in H^1(0,T(U);\mathcal{H})\cap\mathcal{C}([0,T(U)];\mathcal{V})\cap L^2(0,T(U);\mathcal{D}(A))$$

to (2.1), *the number* $T(U)>0$ *is determined by the norm* $\|U\|$.

Now, let $\mathcal{U}_{ad}$ be a closed, bounded and convex subset in $\mathcal{V}$ and let $S>0$ be such that for each $U\in\mathcal{U}_{ad}$, (2.1) has a unique weak solution $Y(U)\in H^1(0,S;\mathcal{H})\cap\mathcal{C}([0,S];\mathcal{V})\cap L^2(0,S;\mathcal{D}(A))$. Thus the problem (P) is obviously formulated as follows:

$$(\overline{\mathrm{P}}) \qquad \text{minimize } J(U),$$

where

$$J(U)=\int_0^S\|Y(U)-Y_d\|^2dt+\gamma\|U\|^2, \quad U\in\mathcal{U}_{ad}.$$

Here, $Y_d=\binom{y_d}{\rho_d}$ is a fixed element of $L^2(0,S;\mathcal{V})$. γ is a positive constant.

We consider the map $U\to Y(U)$ that the initial value U to the corresponding solution $Y(U)$. We show that the map is continuous.

Lemma 2.2. Let $U,V\in\mathcal{U}_{ad}$ and let $Y(U)$ and $Y(V)$ be solutions of (2.1) with respect to U,V, respectively. Then, we have

$$|Y(U)-Y(V)|^2+\int_0^t\|Y(U)(\tau)-Y(V)(\tau)\|^2d\tau\le C|U-V|^2, \qquad 0\le t\le S.$$

PROOF. It is seen that $W=Y(U)-Y(V)$ satisfies

$$\begin{cases}\dfrac{dW(t)}{dt}+AW(t)=F(Y(U))-F(Y(V)), & 0<t\le S,\\ W(0)=U-V.\end{cases} \tag{2.2}$$

Taking the scalar product of the equation of (2.2) with W, we have

$$\frac{1}{2}\frac{d}{dt}|W(t)|^2+\langle AW(t),W(t)\rangle=\langle F(Y(U))-F(Y(V)),W(t)\rangle.$$

From (a.ii) and (f.ii), it follows that

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}|W(t)|^2+\delta\|W(t)\|^2 \qquad (2.3)\\ \le&\eta\|W(t)\|^2+(\|Y(U)\|+\|Y(V)\|+1)\psi_\eta(|Y(U)|+|Y(V)|)|W(t)|\|W(t)\|\\ \le&\frac{\delta}{2}\|W(t)\|^2+C(\|Y(U)\|^2+\|Y(V)\|^2+1)\psi_{\delta/4}(|Y(U)|+|Y(V)|)^2|W(t)|^2.\end{aligned}$$

Therefore, by Gronwall's lemma,

$$|W(t)|^2 \le |W(0)|^2 e^{\int_0^S C(\|Y(U)\|^2+\|Y(V)\|^2+1)\psi_{\delta/4}(|Y(U)|+|Y(V)|)^2 dt}.$$

Using this result in (2.3) and integrating from 0 to t, we obtain the estimate for

$$\int_0^t \|Y(U)(\tau) - Y(V)(\tau)\|^2 d\tau. \quad \square$$

Theorem 2.3. There exists an optimal control $\overline{U} \in \mathcal{U}_{ad}$ for $(\overline{\mathrm{P}})$ such that

$$J(\overline{U}) = \min_{U \in \mathcal{U}_{ad}} J(U).$$

PROOF. Let $\{U_n\} \subset \mathcal{U}_{ad}$ be a minimizing sequence such that

$$\lim_{n\to\infty} J(U_n) = \min_{U \in \mathcal{U}_{ad}} J(U).$$

Since $\{U_n\}$ is bounded in $\mathcal{V}$, we can assume that $U_n \to \overline{U}$ weakly in $\mathcal{V}$ and by the compactness of $\mathcal{V} \hookrightarrow \mathcal{H}$, $U_n \to \overline{U}$ strongly in $\mathcal{H}$. By Lemma 2.2, $Y(U_n) \to Y(\overline{U})$ strongly in $L^2(0, S; \mathcal{V})$. Therefore, $Y(U_n) - Y_d$ is strongly convergent to $Y(\overline{U}) - Y_d$ in $L^2(0, S; \mathcal{V})$, we have

$$\min_{U \in \mathcal{U}_{ad}} J(U) \le J(\overline{U}) \le \liminf_{n\to\infty} J(U_n) = \min_{U \in \mathcal{U}_{ad}} J(U). \quad \square$$

3. Optimality conditions

In this section, we show the optimality conditions for the Problem $(\overline{\mathrm{P}})$. We denote the scalar products in $\mathcal{V}$ and $\mathcal{V}'$ by $\langle \cdot, \cdot \rangle_{\mathcal{V}}$ and $\langle \cdot, \cdot \rangle_{\mathcal{V}'}$, respectively. In order to the optimality conditions, we need some additional assumptions:

$F(\cdot)$ is first-order Fréchet differentiable with the derivative

$$F'(Y)Z = \begin{pmatrix} az + dz(y+\rho-1)(1-2y) + dy(z+w)(1-y) \\ c\nabla\{w(1-2\rho)\nabla\chi(y)\} + c\nabla\{\rho(1-\rho)\nabla(\chi'(y)z)\} \\ -f\alpha\chi'(y)ze^{\alpha\chi(y)}\rho - fe^{\alpha\chi(y)}w - hw \end{pmatrix}, \quad Y = \begin{pmatrix} y \\ \rho \end{pmatrix}, \ Z = \begin{pmatrix} z \\ w \end{pmatrix}.$$

and the following estimates is satisfied.

Lemma. 3.1. (f.iii) For each $\eta > 0$, there exists an increasing continuous function $\mu_\eta : [0, \infty) \to [0, \infty)$ such that

$$|\langle F'(Y)Z, P\rangle| \le \begin{cases} \eta\|Z\|\|P\| + (\|Y\|+1)\mu_\eta(|Y|)|Z|\|P\|, & Y, Z, P \in \mathcal{V}, \\ \eta\|Z\|\|P\| + (\|Y\|+1)\mu_\eta(|Y|)\|Z\||P|, & Y, Z, P \in \mathcal{V}. \end{cases}$$

(f.iv) There exists an increasing continuous function $\nu : [0, \infty) \to [0, \infty)$ such that

$$\|F'(\widetilde{Y})Z - F'(Y)Z\|_* \le C\|Z\|(1 + \|\widetilde{Y}\| + \|Y\|)\nu(|\widetilde{Y}| + |Y|)|\widetilde{Y} - Y|, \quad \widetilde{Y}, Y, Z \in \mathcal{V}.$$

PROOF. The proof is similar to that of [9, Sec. 3]. $\square$

Proposition 3.2 Let (a.i), (a.ii), (f.i), (f.ii), (f.iii), and (f.iv) be satisfied. The mapping $Y : \mathcal{U}_{ad} \to H^1(0,S;\mathcal{V}') \cap \mathcal{C}([0,S];\mathcal{H}) \cap L^2(0,S;\mathcal{V})$ is Gâteaux differentiable with respect to U. For $V \in \mathcal{U}_{ad}$, $Y'(U)V = Z$ is the unique solution in $H^1(0,S;\mathcal{V}') \cap \mathcal{C}([0,S];\mathcal{H}) \cap L^2(0,S;\mathcal{V})$ of the problem

$$\begin{aligned} &\frac{dZ}{dt} + AZ - F'(Y)Z = 0, \quad 0 < t \le S, \\ &Z(0) = V. \end{aligned} \tag{3.1}$$

PROOF. Let $U, V \in \mathcal{U}_{ad}$ and $0 \le h \le 1$. Let Y_h and Y be the solutions of (2.1) corresponding to $U + hV$ and U, respectively. We consider the problem of the following form

$$\begin{aligned} &\frac{d}{dt}\frac{Y_h - Y}{h} + A\frac{Y_h - Y}{h} - \frac{F(Y_h) - F(Y)}{h} = 0, \quad 0 < t \le S, \\ &\frac{Y_h - Y}{h}(0) = V. \end{aligned}$$

On the other hand, we consider the linear problem (3.1). From (a.i), (a.ii), (f.i), (f.ii), and (f.iii), we can easily verify that (3.1) possesses a unique weak solution $Z \in H^1(0,S;\mathcal{V}') \cap \mathcal{C}([0,S];\mathcal{H}) \cap L^2(0,S;\mathcal{V})$ on $[0,S]$ (cf. [3, Chap. XVIII, Theorem 2]. Define $F_h' = \int_0^1 F'(Y + \theta(Y_h - Y))d\theta$. Then $\widetilde{W} = \frac{Y_h - Y}{h} - Z$ satisfies

$$\begin{aligned} &\frac{d\widetilde{W}(t)}{dt} + A\widetilde{W}(t) - F_h'\widetilde{W}(t) = (F_h' - F_0')Z(t), \quad 0 < t \le S, \\ &\widetilde{W}(0) = 0. \end{aligned} \tag{3.2}$$

Taking the scalar product of the equation of (3.2) with $\widetilde{W}$, we obtain that

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}|\widetilde{W}(t)|^2 + \langle A\widetilde{W}(t), \widetilde{W}(t)\rangle \\ =&\langle F_h'\widetilde{W}(t), \widetilde{W}(t)\rangle + \langle (F_h' - F_0')Z(t), \widetilde{W}(t)\rangle \\ \le&\frac{\delta}{2}\|\widetilde{W}(t)\|^2 + (\|Y_h(t)\|^2 + \|Y(t)\|^2 + 1)\tilde{\mu}(|Y_h(t)|^2 + |Y(t)|^2)|\widetilde{W}(t)|^2 \\ &+ \|Z(t)\|^2(\|Y_h(t)\|^2 + \|Y(t)\|^2 + 1)\tilde{\nu}(|Y_h(t)|^2 + |Y(t)|^2)|Y_h(t) - Y(t)|^2, \end{aligned}$$

where $\tilde{\mu}$, $\tilde{\nu} : [0,\infty) \to [0,\infty)$ is some increasing continuous function. Therefore,

$$\begin{aligned} &|\widetilde{W}(t)|^2 + \delta\int_0^t \|\widetilde{W}(s)\|^2 ds \\ \le& \int_0^t (\|Y_h(s)\|^2 + \|Y(s)\|^2 + 1)\tilde{\mu}(|Y_h|^2 + |Y|^2)|\widetilde{W}(s)|^2 ds + \|Y_h(t) - Y(t)\|^2_{L^\infty(0,S;\mathcal{H})} \\ &\times \int_0^t (\|Y_h(s)\|^2 + \|Y(s)\|^2 + 1)\tilde{\nu}(|Y_h(s)|^2 + |Y(s)|^2)\|Z(s)\|^2 ds. \end{aligned}$$

Using $Y_h, Y \in \mathcal{C}([0,S];\mathcal{V})$ and Gronwall's Lemma,

$$|\widetilde{W}(t)|^2 + \delta\int_0^t \|\widetilde{W}(s)\|^2 ds \le C\|Y_h(t) - Y(t)\|^2_{L^\infty(0,S;\mathcal{H})}\|Z\|_{L^2(0,S;\mathcal{V})}.$$

Since $Y_h \to Y$ strongly in $\mathcal{C}([0,S];\mathcal{H})$, it follows that $\frac{Y_h - Y}{h}$ is strongly convergent to Z in $H^1(0,S;\mathcal{V}') \cap \mathcal{C}([0,S];\mathcal{H}) \cap L^2(0,S;\mathcal{V})$. □

Theorem 3.3. Let $\overline{U}$ be an optimal control of $(\overline{\mathrm{P}})$ and let $\overline{Y} \in H^1(0,S;\mathcal{H}) \cap \mathcal{C}([0,S];\mathcal{V}) \cap L^2(0,S;\mathcal{D}(A))$ be the optimal state, that is $\overline{Y}$ is the solution to (2.1) with the control $\overline{U}$. Then, there exists a unique solution $P \in H^1(0,S;\mathcal{V}') \cap \mathcal{C}([0,S];\mathcal{H}) \cap L^2(0,S;\mathcal{V})$ to the linear problem

$$-\frac{dP}{dt} + AP - F'(\overline{Y})^* P = \Lambda(\overline{Y} - Y_d), \quad 0 \le t < S, \qquad (3.3)$$
$$P(S) = 0$$

in $\mathcal{V}'$, where $\Lambda : \mathcal{V} \to \mathcal{V}'$ is a canonical isomorphism; moreover, $\overline{U}$ satisfy

$$\langle \frac{1}{\gamma} P(0) + \Lambda\overline{U}, V - \overline{U}\rangle \ge 0 \qquad \text{for all } V \in \mathcal{U}_{ad}.$$

PROOF. Since J is Gateaux differentiable at $\overline{U}$ and $\mathcal{U}_{ad}$ is convex, it is seen that

$$J'(\overline{U})(V - \overline{U}) \ge 0 \qquad \text{for all } V \in \mathcal{U}_{ad}.$$

On the other hand, we verify that

$$J'(\overline{U})(V - \overline{U}) = \int_0^S \langle Y(\overline{U}) - Y_d, Z\rangle_{\mathcal{V}} dt \; + \; \gamma\langle \overline{U}, V - \overline{U}\rangle_{\mathcal{V}} \qquad (3.4)$$

with $Z = Y'(\overline{U})(V - \overline{U})$. Let P be the unique solution of (3.3) in $H^1(0,S;\mathcal{V}') \cap \mathcal{C}([0,S];\mathcal{H}) \cap L^2(0,S;\mathcal{V})$. From (a.i), (a.ii), and (f.iii), we can guarantee that such a solution P exists on $[0,S]$ (cf. [3, Chap. XVIII, Theorem 2]). Thus, in view of Proposition 3.2 the first integral in the right hand side of (3.4) is shown to be

$$\int_0^S \langle Y(\overline{U}) - Y_d, Z\rangle_{\mathcal{V}} dt = \int_0^S \langle \Lambda(Y(\overline{U}) - Y_d), Z\rangle dt$$
$$= \int_0^S \langle -\frac{dP}{dt} + AP - F'(\overline{Y})^* P, Z\rangle dt = \langle P(0), V - \overline{U}\rangle.$$

Hence,

$$\langle \frac{1}{\gamma} P(0) + \Lambda\overline{U}, V - \overline{U}\rangle \ge 0 \qquad \text{for all } V \in \mathcal{U}_{ad}. \; \square$$

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피흡착질에 의하여 유도된 상전이 모델에 대한 초기치 제어문제

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요 약

본 논문에서는 피흡착질에 의하여 유도된 상전이 모델에 대한 초기치 제어 문제를 다루고 있다. 구체적으로, 적절한 비용함수를 선택하여 그 함수를 최소화하는 최적제어의 존재성을 보였다. 또한, 최적제어이론에서 중요한 문제중의 하나인 최적제어가 만족해야할 최적성의 필요조건을 얻었다.