대규모 시스템의 계층적 최적제어 및 수질오염제어에의 그 응용

김 경 연*·감 상 규**

Hierarchical Optimal Control of Large-Scale Systems and Its Applications to River Pollution Control

*Kyung-youn Kim*and Sang-kyu Kam***

요 약

강의 수질오염제어에 적용하기 위하여, 상호작용 예측방법을 이용하여 시간지연이 있는/없는 대규모 시스템의 최적제어를 위한 계층제어 기법을 일관성있게 기술한다. 미리 결정된 공칭입력을 도입함으로써 서보메카니즘 문제를 조정기 문제로 변형하고, 변형된 문제에 대한 최적해를 계층적 방법으로 구한다. 특히 시간지연이 없는 모델에 대해서는, 폐회로 제어시스템을 구성하기 위하여 모든 초기조건에 대해 최적인 이득행렬과 보상벡터를 구한다. 수질오염모델에 대한 컴퓨터 모사를 통하여 제안된 앨고리듬의 타당성을 확인한다.

Abstract

A hierarchical technique, which is based on the interaction prediction principle, is described in a unified manner for the optimal control of large-scale systems with/without time-delays to apply river pollution control. The optimal servomechanism problem is transformed to the regulator problem by introducing a predetermined nominal input into the performance index and the optimal solution to the transformed problem is obtained in a hierarchical manner. Especially in the case of no-delay model, the feedback gain matrix and the compensation vector which are optimal for any initial conditions can be obtained to construct closed-loop control. Compuer simulations for the river pollution models are provided to demonstrate the validity of the proposed algorithm.

*제주대학교 공과대학 전자공학과(Dept. of Electronic Engineering, College of Engineering)
**제주대학교 해양과학대학 해양환경공학과(Dept. of Marine Environmental Engineering, College of Ocean Sciences)

Ⅰ. Introduction

For nearly two decades, there has been considerable interest in the development of hierarchical control of large-scale complex systems[1-5]. The advent of parallel processing technology and emphasis on fault-tolerant system design are additional factors motivating such development. Basically, the hierarchical control technique is composed of decomposition and coordination processes. In decomposition process the large-scale problem is divided into a number of smaller subproblems which can be solved independently each other at a lower level. On a upper level, the coordination variables are updated successively to force the lower level solutions to be the optimal solution of the overall problem. Therefore, through multilevel methodologies, a large-scale control system's complexity can be relaxed by solving decomposed subproblems which are of smaller dimensions.

Although various hierarchical multilevel control techniques for the large-scale systems have been reported in the literature[6,7], the control obtained by these methods is open-loop in nature so that it is necessary to recalculate it every time an unknown disturbance changes the initial state of the system. To get around the computational difficulties which are associated with computational time and storage space, Singh et al.[8-11] have proposed a promising hierarchical algorithm by using interaction prediction method. This algorithm is found to be superior to other multilevel methods for a certain class of optimization problems. On the upper-level, it has more rapid convergence rate and fewer operations than other coordination rules such as linear search algorithm[7]. But it also has a disadvantage that dimension of the given system has to be increased for the optimal control of time-delay systems.

In this paper, we describe an efficient hierarchical optimal control method for large-scale systems with/without time-delays in states and inputs to apply river pollution control. The optimal servomechanism problem is transformed to the regulator problem with constant input by introducing a predetermined nominal input into the performance index and the optimal solution to the transformed problem is obtained in a hierarchical manner. The steady-state error which is defined as the difference between the target state and actual state in steady-state is derived analytically. In no-delay case, the feedback gain matrix and the compensation vector which are optimal for all the initial conditions are calculated so that eventual on-line computation is minimal.

The rest is organized as follows. In section 2, the optimal control problem of large-scale systems is formulated and a hierarchical optimization technique is described in section 3. Section 4 provides steady-state considerations in steady-state error and closed-loop control. Simulation results for the river pollution model are provided in section 5 and the conclusion is described in the final section.

Ⅱ. Problem Formulation

Consider the following linear quadratic(LQ) tracking problem of large-scale system with time-delays in states and inputs:

$$x(k+1) = \sum_{l=0}^{\theta_x} A_l x(k-l) + \sum_{l=0}^{\theta_u} B_l u(k-l) + c \tag{1}$$

$$J = \frac{1}{2} \sum_{k=0}^{k_f-1} \{ \| x(k) - x^d \|_Q^2 + \| u(k) - u^n \|_R^2 \} \tag{2}$$

with initial conditions

$$x(k) = \phi_x(k), \quad -\theta_x \le k \le 0 \tag{3a}$$

$$u(k) = \phi_u(k), \quad -\theta_u \le k < 0 \tag{3b}$$

where $A_l (l=0,1,2,\cdots,\theta_x) \in R^{n \times n}$ is a system matrix, $B_l (l=0,1,2,\cdots,\theta_u) \in R^{n \times m}$ is an input matrix, $c \in R^{n \times 1}$ is a constant input vector, θ_x is a maximum time-delay in states, θ_u is a maximum time-delay in control inputs, $Q \in R^{n \times n}$ is a state weighting matrix, $R \in R^{m \times m}$ is an input weighting matrix, $x^d \in R^{n \times 1}$ is a constant desired or target value of state vector and $u^n \in R^{m \times 1}$ is a predetermined nominal control input, which will be discussed in section 4. It is assumed that Q and R are positive semi-definite and positive definite block diagonal matrix, respectively. Here, the optimal control problem is to find a control law which causes the state vector of the system (1) to follow a desired value that minimizes the performance index (2).

Define a new state and control vector as follows:

$$z(k) \equiv x(k) - x^d \tag{4a}$$

$$v(k) \equiv u(k) - u^n \tag{4b}$$

Then we can obtain the following transformed regulator problem from the above optimal servomechanism problem.

$$z(k+1) = \sum_{l=0}^{\theta_x} A_l z(k-l) + \sum_{l=0}^{\theta_u} B_l v(k-l) + c^b \tag{5}$$

$$J = \frac{1}{2} \sum_{k=0}^{k_f-1} \{ \| z(k) \|_Q^2 + \| v(k) \|_R^2 \} \tag{6}$$

where

$$c^b = \left[\sum_{l=0}^{\theta_x} A_l - I_n \right] x^d + \left[\sum_{l=0}^{\theta_u} B_l \right] u^n + c \tag{7}$$

with initial conditions

$$z(k) = \phi_x(k) - x^d, \quad -\theta_x \le k \le 0 \tag{8a}$$

$$v(k) = \phi_u(k) - u^n, \quad -\theta_u \le k < 0 \tag{8b}$$

The centralized optimal control is prohibitive to the above large-scale system due to computational difficulties which are associated with computational time and storage space. To get around the computational difficulties, we develope a hierarchical optimal control technique based on interaction prediction method.

Ⅲ. Hierarchical Optimization

Let's decompose the above centralized optimal regulator problem into a number of smaller subproblems to obtain the optimal solution in a hierarchical manner. The i-th subproblem is expressed as:

$$z_i(k+1) = A_{ii} z_i(k) + B_{ii} v_i(k) + c_i^b + h_i(k) \tag{9}$$

$$h_i(k) = \sum_{(j \ne i, \text{ if } l=0)}^{N} \left\{ \sum_{l=0}^{\theta_x} L_{ijl} z_j(k-l) + \sum_{l=0}^{\theta_u} M_{ijl} v_j(k-l) \right\} \tag{10}$$

$$J_i = \frac{1}{2} \sum_{k=0}^{k_f-1} \{ \| z_i(k) \|_{Q_i}^2 + \| v_i(k) \|_{R_i}^2 \} \tag{11}$$

with initial conditions

$$z_i(k) = \phi_{x_i}(k) - x_i^d, \quad -\theta_x \le k \le 0 \tag{12a}$$

$$v_i(k) = \phi_u(k) - u_i^*, \quad -\theta_u \le k < 0 \tag{12b}$$

where $h_i(k) \in R^{n_i \times 1}$ consists of interaction inputs which come in from the other subsystems and time-delayed states of the i-th subsystem, $L_{ijl} \in R^{n_i \times n_j}$ is a coupling matrix of states, $M_{ijl} \in R^{n_i \times m_j}$ is a coupling matrix of control inputs and N is the number of the interconnected subsystems which comprise the overall system.

Now, we use the interaction prediction method which is attractive due to simple upper-level algorithm and fast convergence rate. Basically, the interaction prediction method is composed of two levels. The optimal solutions of decomposed subproblems are obtained at lower-level and the coordination vector is updated at upper-level to force the independent lower-level solutions to be the optimal solution of the overall system. Firstly, consider the lower-level problem to find the optimal solutions for the decomposed subproblems. The Hamiltonian function for the i-th subsystem can be written as:

$$\begin{aligned} H_i = & \frac{1}{2}\{ \| z_i(k) \|^2_{Q_i} + \| v_i(k) \|^2_{R_i} \} + \gamma_i^T(k) h_i(k) \\ & - \sum_{(j \ne i,\ \text{if } l=0)}^{N} \left\{ \sum_{l=0}^{\theta_x} \gamma_j^T(k+l) L_{jil}\, z_i(k) + \sum_{l=0}^{\theta_u} \gamma_j^T(k+l) M_{jil}\, v_i(k) \right\} \\ & + q_i^T(k+1) \{ A_{ii}\, z_i(k) + B_{ii}\, v_i(k) + c_i^o + h_i(k) \} \end{aligned} \tag{13}$$

where $\gamma_i(k) \in R^{n_i \times 1}$ and $q_i(k) \in R^{n_i \times 1}$ are Lagrange multiplier and costate vector of i-th subsystem, respectively. From (13) the necessary conditions for optimality are obtained as[11]:

$$z_i(k+1) = A_{ii} z_i(k) + B_{ii} v_i(k) + c_i^o + h_i(k) \tag{14}$$

$$z_i(0) = \phi_{x_i}(0) - x_i^d \tag{15}$$

$$v_i(k) = -R_i^{-1}[\, B_{ii}^T q_i(k+1) - \sum_{(j \ne i,\ \text{if } l=0)}^{N} \sum_{l=0}^{\theta_u} M_{jil}^T \gamma_j(k+l)] \tag{16}$$

$$\gamma_i(k) = 0, \quad (k \ge k_f) \tag{17}$$

$$q_i(k) = Q_i z_i(k) + A_{ii}^T q_i(k+1) - \sum_{(j \ne i,\ \text{if } l=0)}^{N} \sum_{l=0}^{\theta_x} L_{jil}^T \gamma_j(k+l) \tag{18}$$

$$q_i(k_f) = 0 \tag{19}$$

Next, consider the upper-level coordination rule in order to force the lower-level solutions to be the optimal solution of the overall system. For this purpose, the additively separable Lagrangian function can be written as:

$$\begin{aligned} L = & \sum_{i=1}^{N} \sum_{k=0}^{k_f-1} [\, \frac{1}{2}\{ \| z_i(k) \|^2_{Q_i} + \| v_i(k) \|^2_{R_i} \} + \gamma_i^T(k) h_i(k) \\ & - \sum_{(j \ne i,\ \text{if } l=0)}^{N} \gamma_i^T(k) \left\{ \sum_{l=0}^{\theta_x} L_{ijl}\, z_j(k-l) + \sum_{l=0}^{\theta_u} M_{ijl}\, v_j(k-l) \right\} \\ & + q_i^T(k+1) \{ A_{ii}\, z_i(k) + B_{ii}\, v_i(k) + c_i^o + h_i(k) - z_i(k+1) \}] \end{aligned} \tag{20}$$

A necessary condition for the overall optimum is given as[11]:

$$\frac{\partial L}{\partial h_i(k)} = 0 \text{ and } \frac{\partial L}{\partial \gamma_i(k)} = 0 \tag{21}$$

Then the coordination rule at the upper-level from iteration L to L+1 is obtained by

$$\begin{bmatrix} \gamma_i(k) \\ h_i(k) \end{bmatrix} = \begin{bmatrix} -q_i(k+1) \\ \sum_{(j \ne i,\ \text{if } l=0)}^{N} \left\{ \sum_{l=0}^{\theta_x} L_{ijl}\, z_j(k-l) + \sum_{l=0}^{\theta_u} M_{ijl}\, v_j(k-l) \right\} \end{bmatrix} \tag{22}$$

Now, a step-by-step computational procedure to obtain optimal control law is summarized as follows.

step 1: At the upper-level, set L=1 and predict initial values for $\gamma_i(k)$ and $h_i(k)$ (i=1,2,...,N, k=0,1,...,k_f-1). Then pass them down to the lower-level.

step 2: At the lower-level, solve the independent necessary conditions for optimality (14)-(19) by using $\gamma_i(k)$ and $h_i(k)$ passed from upper-level. Then send $Z_i(k)$, $v_i(k)$ and $q_i(k)$ (i=1,2,...,N, k=0,1,...,k_f-

1) to the upper-level.

step 3: At the upper-level, check the convergence of (22). i.e., whether their errors are within the predetermined error bounds, ϵ. If not, update $\gamma_i(k)$ and $h_i(k)$ from (22) by using $Z_i(k)$, $v_i(k)$ and $q_i(k)$ passed from the lower-level. Then set L=L+1 and go to step 2.

step 4: If step 3 is converged, calculate the optimal control law and state trajectory from (4a) and (4b), respectively.

Ⅳ. Steady-State Considerations

If the final time k_f is large enough for the system to reach a steady-state, we can derive the steady-state error analytically and obtain closed-loop control law. Firstly, consider the steady-state error.

Theorem 1: If the proposed hierarchical algorithm in section 3 for the optimal control of large-scale system (1) through (3) converges, the steady-state error is given as:

$$e_{ss} = -\left\{ I_n - \sum_{l=0}^{\theta_x} A_l + \left(\sum_{l=0}^{\theta_u} B_l\right) R^{-1} \left(\sum_{l=0}^{\theta_u} B_l^T\right) \left(I_n - \sum_{l=0}^{\theta_x} A_l^T\right)^{-1} Q \right\}^{-1} c^d \tag{23}$$

Proof of Theorem 1

If the algorithm is converged, the left-hand side of (22) is equal to the right-hand side. Hence we obtain the following integrated expressions:

$$z(k+1) = \sum_{l=0}^{\theta_x} A_l z(k-l) + \sum_{l=0}^{\theta_u} B_l v(k-l) + c^d \tag{24}$$

$$v(k) = -R^{-1} \sum_{l=0}^{\theta_u} B_l^T q(k+l+1) \tag{25}$$

$$q(k) = Qz(k) + \sum_{l=0}^{\theta_x} A_l^T q(k+l+1) \tag{26}$$

Since z(k), v(k) and q(k) are constant vectors at steady-state, we have

$$z_s = \sum_{l=0}^{\theta_x} A_l z_s + \sum_{l=0}^{\theta_u} B_l v_s + c^d \tag{27}$$

$$v_s = -R^{-1} \sum_{l=0}^{\theta_u} B_l^T q_s \tag{28}$$

$$q_s = Qz_s + \sum_{l=0}^{\theta_x} A_l^T q_s \tag{29}$$

where the subscript s denotes steady-state. Combining (27), (28) and (29), we obtain

$$\left[I_n - \sum_{l=0}^{\theta_x} A_l \right] z_s = -\left[\sum_{l=0}^{\theta_u} B_l \right] R^{-1} \left[\sum_{l=0}^{\theta_u} B_l^T \right] \left[I_n - \sum_{l=0}^{\theta_x} A_l^T \right]^{-1} Q z_s + c^d \tag{30}$$

Define the steady-state error as the difference between the target state and actual state in steady state:

$$e_{ss} \equiv x^d - x_s \tag{31}$$

Taking into account (4a) and (31), we obtain (23) from (30). This completes the proof.

Remark 1:

(a) The quantity inside the braces on the right-hand side of (23) is nonsingular if the inverse of $\left[I_n - \sum_{l=0}^{\theta_x} A_l \right]$ exists.

(b) The necessary and sufficient condition for zero steady-state error is that a vector $\left[I_n - \sum_{l=0}^{\theta_x} A_l \right] x^d - c$ belongs to the col-u mn space of a matrix $\sum_{l=0}^{\theta_u} B_l$.

(c) The steady-state error can be obtained from the given state equation and performance index without solving the optimal control algorithm.

Remark 2:

(a) If the necessary and sufficient condition for zero steady-state error is satisfied, the nominal control input u^n is obtained by

$$u^n = \left\{ [\sum_{l=o}^{\theta_u} B_l^T][\sum_{l=o}^{\theta_u} B_l] \right\}^{-1} [\sum_{l=o}^{\theta_u} B_l^T] \left\{ [I_n - \sum_{l=o}^{\theta_x} A_l] x^d - c \right\} \tag{32}$$

(b) If the condition is not satisfied, the nominal control input obtained from (32) is a approximate least-square solution for C^p=0 In this case the steady-state error is given as

$$e_{ss} = \left\{ I_n - (\sum_{l=0}^{\theta_x} A_l) + (\sum_{l=0}^{\theta_u} B_l) R^{-1} (\sum_{l=0}^{\theta_u} B_l^T)(I_n - \sum_{l=0}^{\theta_x} A_l^T)^{-1} Q \right\}^{-1} \{ I_n - (\sum_{l=0}^{\theta_u} B_l)(\sum_{l=0}^{\theta_u} B_l^T \sum_{l=0}^{\theta_u} B_l)^{-1} (\sum_{l=0}^{\theta_u} B_l^T) \} \left\{ (I_n - \sum_{l=0}^{\theta_x} A_l) x^d - c] \right\} \tag{33}$$

Next, consider the closed-loop control law which can be obtained as follows for the no-delay model(l=0)[12].

$$u(k) = Gx(k) + d \tag{34}$$

where G and d are the constant feedback gain matrix and the compensation vector, respectively. The procedure to obtain G and d from the hierarchical algorithm can be summarized as follows.

In case of c^p=o the feedback gain matrix is obtained by

$$G = V_1^* Z_1^{*-1} \tag{35}$$

where V_1^* and Z_1^* are the optimal solutions obtained from the hierarchical algorithm, which are defined by

$$V_1^* = [\ v(0)\ v(1) \cdots v(n-1)] \tag{36a}$$

$$Z_1^* = [\ z(0)\ z(1) \cdots z(n-1)] \tag{36b}$$

Then, from(4a) and (4b) we can obtain the compensation vector as:

$$d = -Gx^d + u^n \tag{37}$$

In case $c^p \neq$o we can obtain the feedback gain matrix as:

$$G = V_2^* Z_2^{*-1} \tag{38}$$

where

$$V_2^* \equiv [\ v(0)-v(n)\ v(1)-v(n) \cdots v(n-1)-v(n)] \tag{39a}$$

$$Z_2^* \equiv [\ z(0)-z(n)\ z(1)-z(n) \cdots z(n-1)-z(n)] \tag{39b}$$

Also, from(4a) and (4b), the compensation vector is given by

$$d = -Gx^d + u^n + d_1 \tag{40}$$

where d_1 is defined as:

$$d_1 = v(o) - Gz(0) \tag{41}$$

Ⅴ. Numerical Examples

To illustrate the proposed algorithm, we consider the following two examples.

Ex. 1: River pollution model with no time-delay[13]

The numerical values for the model are $N=2$, $n_i=2$, $m_i=1(i=1, 2)$, $\theta_x=0$, $\theta_u=0$,

$$A_{ii} = \begin{bmatrix} 0.18 & 0.0 \\ -0.25 & 0.27 \end{bmatrix},\quad B_{ii} = \begin{bmatrix} -2.0 \\ 0.0 \end{bmatrix},\ (i=1,2)$$

$$c_1 = \begin{bmatrix} 4.5 \\ 6.15 \end{bmatrix},\quad c_2 = \begin{bmatrix} 2.0 \\ 2.65 \end{bmatrix}$$

$$L_{210} = \begin{bmatrix} 0.55 & 0.0 \\ 0.0 & 0.55 \end{bmatrix},\ L_{120} = 0,$$

$$M_{120} = M_{210} = 0\ Q_i = I_2,\ R_i = 100,\ \varepsilon = 10^{-5} \text{ and } k_f = 20$$

which is large enough for the system to reach a steady-state.

Simulations are carried out for the

following two cases:

Case 1: The necessary and sufficient condition for zero steady-state error is satisfied:

$$x_1^d = [\ 4.16\ 7.0]^T \text{ and } x_2^d = [\ 5.56\ 7.0]^T.$$

Case 2: The necessary and sufficient condition for zero steady-state is not satisfied:

$$x_1^d = [\ 5.0\ 7.0]^T \text{ and } x_2^d = [\ 5.0\ 7.0]^T.$$

The proposed hierarchical algorithm was converged after 10 iterations and the feedback gain matrix for two cases is obtained as follows:

$$G = \begin{bmatrix} .0074610 & -.0011551 & .0005578 & -.0001175 \\ .0129503 & -.0017040 & .0041831 & -.0003941 \end{bmatrix}$$

And the compensation vector is given as follows:

$$d = [\ 0.5192195\ \ -0.1980404]^T : \text{Case 1}$$

$$d = [\ 0.1709616\ \ -0.2537002]^T : \text{Case 2}$$

The square root steady-state error for Case 1 is always zero irrespective of weighting matrices. And the steady-state error for Case 2 is 0.0812 ($R=50I_2$), 0.0821($R=100I_2$) and 0.0831($R=500I_2$). In general, it is noted that an increase in R reduces the steady-state error.

In addition, the optimal trajectories of state variables for the Case 1 are shown in Fig.1. The results obtained from the hierarchical algorithm are nearly identical to those of the centralized optimal control which is omitted here.

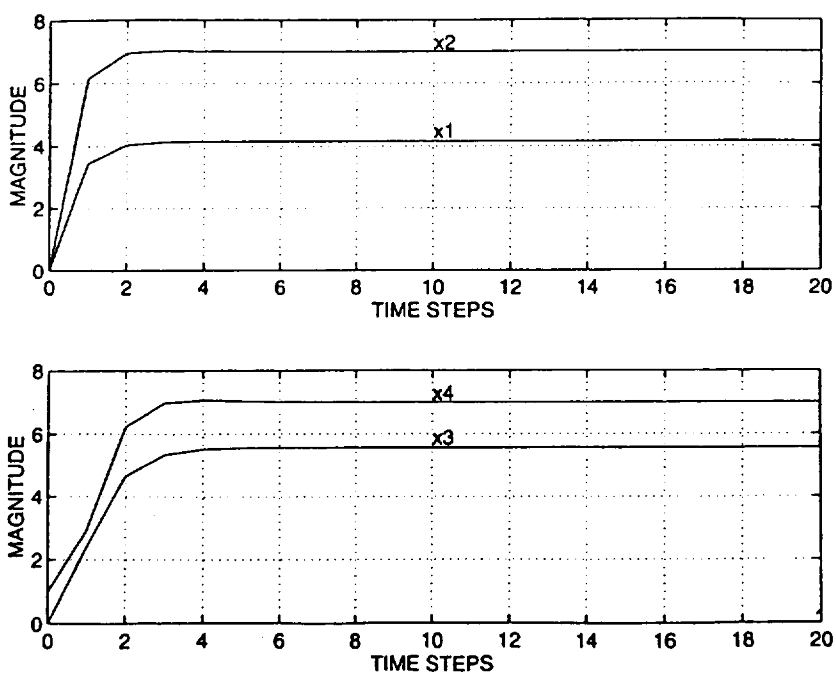

Fig. 1. Optimal trajectories of state variables

Ex. 2: River pollution model with distributed time-delay[12)]

The numerical values for the model are N=3, $n_i=2$, $m_i=1$(i=1,2,3), $\theta x=2$, $\theta u=0$,

$$A_{ii} = \begin{bmatrix} 0.18 & 0.0 \\ -0.25 & 0.27 \end{bmatrix}, \quad B_{ii} = \begin{bmatrix} -2.0 \\ 0.0 \end{bmatrix}, \ (i=1,2,3)$$

$$c_1 = \begin{bmatrix} 4.5 \\ 6.15 \end{bmatrix}, \quad c_2 = \begin{bmatrix} 2.0 \\ 2.65 \end{bmatrix}, \quad c_3 = \begin{bmatrix} 2.0 \\ 2.65 \end{bmatrix}$$

$$L_{210} = L_{330} = L_{212} = L_{322} = \begin{bmatrix} 0.0825 & 0.0 \\ 0.0 & 0.0825 \end{bmatrix}, \ L_{211} = L_{321} = \begin{bmatrix} 0.385 & 0.0 \\ 0.0 & 0.385 \end{bmatrix}$$

$Q_i = I_2$, $R_i = 100$, $\epsilon = 10^{-5}$, $k_f = 30$ and the other values of L_{ijl} and M_{ij} are 0.

Simulations are also carried out for the following two cases.

Case 1: $x_1^d = [\ 4.16\ 7.0]^T$ and $x_2^d = x_3^d = [\ 5.56\ 7.0]^T$.
Case 2: $x_1^d = [\ 5.0\ 7.0]^T$ and $x_2^d = x_3^d = [\ 5.0\ 7.0]^T$.

The proposed hierarchical algorithm was converged after 15 iterations and the square root value of steady-state error for case 1 is zero as expected and for case 2 is 0.1123.

The simulation results for two examples show that the proposed algorithm has comparatively fast convergence rate and the steady-state error is exactly consistent with Theorem 1. Therefore, we can obtain the steady-state error from the given state equation and performance index without solving the hierarchical optimal control algorithm.

Ⅶ. Conclusion

A two-level hierarchical technique, which is based on the interaction prediction principle, is described in a unified manner for the optimal control of large-scale systems with/without time-delays to apply river pollution control. The optimal servomechanism problem is transformed to the regulator problem by introducing a predetermined nominal input into the performance index and the optimal solution to the transformed problem is obtained in a hierarchical manner. The steady-state error for the proposed method is derived analytically. In addition, the feedback gain matrix and the compensation vector which are optimal for any initial conditions can be obtained for no-delay model. Computer simulation results for river pollution models show that the proposed hierarchical algorithm has comparatively fast convergence rate and that the steady-state error can be calculated from the state equation and performance index.

Further work is currently being carried out to utilize these hierarchical algorithm in a real field, such as road traffic control and communication routing control.

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