科學教育, 12, 177~192, 1995
Journal of Science Education
12, 177~192, 1995

브라디온과 초광속 파이온으로 구성된 핵자의 구조

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The Structure of the Nucleons as Compound of a Bradyon and a Tachyonic Pion

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Abstract

The Structure of nucleons as compound of a bradyon and a tachyonic pion is discussed. Assuming that mass and charge density distributions of the tachyonic pion in its rest frame are represented by exponential functions, we can calculate the spin, magnetic moment, charge and magnetic moment root mean square radii of nucleons. If it were also supposed that the excited states of the nucleons are caused by the finite speed of the tachyonic pion, we can calculate the masses of nucleon resonances (N) and delta resonances (Δ) by the supposed canonical momentum quantization condition.

Ⅰ. Introduction

It was suggested long ago that the virtual cloud of the hadrons are made

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of tachyons[1] and the unstable resonance particles might be composed of bradyon particles and tachyons[2]. From the known fact that a free bradyon and a free tachyon can trap each other in a relativistically invariant way, Corben built up a relativistically invariant bootstrap[3]. But he did not completely explain the dynamics of the resonances. So we further develop the theory and wish to explain the properties of Nucleon's spin, magnetic moments and resonances.

From the usual expression for the energy-momentum of relativistic particle, we have

$$E = \frac{m_0}{(1-v^2)^{\frac{1}{2}}}, \quad |p| = \frac{m_0 v}{(1-v^2)^{\frac{1}{2}}} . \tag{1}$$

Hence E and $|p|$ would become imaginary if we take $v > 1$. We note that the rest mass m_0 which appear in eq.(1) cannot be measured directly even for slow particles. Since only energy and momentum are measureable, we are therefore free to hypothesize particles for which speed $v > 1$ always and m_0 is imaginary quantity [4]

$$m_0 = i m_1 \quad (m_1 \ real) . \tag{2}$$

In this case, the energy and momentum will remain real quantities, satisfying

$$E = \frac{m_1}{(v^2-1)^{\frac{1}{2}}}, \quad |p| = \frac{m_1 v}{(v^2-1)^{\frac{1}{2}}} , \tag{3}$$

and $E^2 - p^2 = -m_1^2$. We note that $|p| > E$. and that the velocity is defined by $v = \frac{|p|}{E} > 1$. For this tachyon the value $v = \infty$ is allowed, and at this infinite speed tachyons carry constant momentum m_1 but no energy.

Such an infinite speed transcendent state of a tachyon is analogous to quantum mechanical state at rest, in which a definite value of momentum implies total uncertainty of position. Since quantum mechanics deals largely with phase velocity, a tachyon is more acceptable in quantum theory than in classical theory[3]. A tachyon with infinite velocity is described by a wavefunction that is periodic in space and independent of time, but a bradyon (time-like particle) at rest is described by a wavefunction that is periodic in time and independent of position.

Actually Corben considered two particles satisfying the Klein-Gorden equations[3,5]

$$D_\mu^{(b)} D_\mu^{(b)} \psi_b = m_0^2 \psi_b, \quad D_\mu^{(t)} D_\mu^{(t)} \psi_t = -m_1^2 \psi_t, \tag{4}$$

with $D_\mu^{(b)} = \partial_\mu - ieA_0$, and $D_\mu^{(t)} = \partial_\mu - ie_1 A_\mu$, $m_0 \geq m_1$. The first Klein-Gorden equation describes a bradyon of mass m_0 and charge e_0, but the second describes a tachyon with imaginary part rest mass(real) m_1 and charge e_1. We postulate the auxiliary condition between these time-like and space-like states:

$$D_\mu^{(b)} \psi_b D_\mu^{(t)} \psi_t = 0 . \tag{5}$$

If $\psi = \psi_b \psi_t$, it follows that

$$D_\mu D_\mu \psi = m^2 \psi \; , \tag{6}$$

where $m^2 = m_0^2 - m_1^2 \geq 0, D_\mu = \partial_\mu - ieA_\mu$ and $e = e_0 + e_1$. Applied to the eigenstates of energy and momentum, eq.(5) implies

$$p_\mu^{(b)} p_\mu^{(t)} = 0 \leftrightarrow p^{(b)} \perp p^{(t)}. \tag{7}$$

Therefore, if a free bradyon with mass m_0 and free tachyon with imaginary part rest mass m_1 have infinite relative speed, they can trap each other in a relativistically invariant way.

Let us now assume that a tachyon which is revolving along a circumference around the bradyon is bound by a repulsive force[6]. It reaches minimal (potential) energy when its speed diverges, so that condition eq.(7) is satisfied. In Such a case one may consider the bradyon-tachyon compound as a couple of two free particles[7].

Let us also suppose that "spperluminal Lorentz transformation" exist in four dimensions. Then the electric four current density and the other four vectors must transform in the same way as four momentum[3]. Therefore, if we assume that tachyonic electric charge is $\rho_{ch}(r')$ in its rest frame, similar to the mechanical momentum, the electric current density $j(r)$ becomes constant as $v \to \infty$

$$j(r) = \rho_{ch}(r'), \quad when \; v = \infty \; . \tag{8}$$

We have already seen similar result with momentum: $|p| = m_1$, when $v = \infty$. The superluminal transformation also transforms the Klein-Gorden equation according to

$$\Box\psi = m^2\psi \rightarrow \Box'\psi' = -m^2\psi' . \tag{9}$$

Thus to every bradyon we could expect the existence of a tachyon with the same mass and internal quantum numbers[3].

To study the shape of these tachyons, we consider a particle that its surface is spherical when at rest, and it becomes ellipsoid when subluminal, and when superluminal such a surface becomes a two sheeted hyperboloid. If it is point-like, it becomes, when superlumional, a double cone infinitely extended in space and rigidly moving with the tachyon speed v. Therefore, tachyons appear more similar to fields than to points and it would be desirable to find out the space-time function yielding the density distribution of a tachyon[8-9].

Ⅱ. Model

We assume that a trapped tachyon is rotating along circles around the bradyon at the center with infinite relative speed inside the nucleon. We also suppose that a free bradyon's rest mass m_0 is 939 MeV, charge e_0 equals to 0 ($+e$) and a free tachyon's mass m_1 is 139.56 MeV, charge e_1 is $+e$ ($-e$) for the proton (neutron). Then the proton's (neutron's) rest mass will be 928.57 MeV and charge $+e$ (0). Although these nucleon's rest mass is not unexpected quantity, the total energy of the compound particle in the bradyon rest frame is the same as the observed rest mass energy (939 MeV) of the nucleons. Furthermore, the fact that there exist none-zero constant momentum and charge current density of the infinite speed tachyon inside the nucleon will enable us to calculate the spin and magnetic moment of the nucleons.

It is known, through electron scattering experiment, that nucleons are not point particles but have fuzzy boundary[10]. It is expected that the magnetization is also distributed over the volume of the nucleon[11], and the charge distribution function can be put as follows[12,13]

$$\rho(r) = \rho(0)\ e^{-\frac{r}{a}},\ where\ a = constant, \tag{10}$$

with the dipole fit, the root mean square radius $\langle r_E^2 \rangle^{\frac{1}{2}}$ and magnetic moment $\langle r_M^2 \rangle^{\frac{1}{2}}$ are [13,14]

$$\langle r_E^2(proton)\rangle^{\frac{1}{2}} \cong \langle r_M^2(proton)\rangle^{\frac{1}{2}} \cong \langle r_M^2(\text{neutron})\rangle^{\frac{1}{2}} \cong 0.84\ fm, \tag{11}$$

but very small or vanishing neutron charge radius is mystery.

Simiarly to these exponential function model of the nucleon charge distribution function, we consider the revolving tachyon (we call it tachyonic π^+ for proton and π^- for neutron hereafter) charge density distribution function $\rho_{p,ch}(r')$ (normalize to unit volume) of protons and $\rho_{n,ch}(r')$of neutrons in its rest frame as follows

$$\begin{aligned} \rho_{p,ch}(r') &= -\lambda\frac{\alpha^3}{8\pi}e^{-\alpha r'} + (1+\lambda)\frac{\beta^3}{8\pi}e^{-\beta r'}, \\ \rho_{n,ch}(r') &= x[\frac{\alpha^3}{8\pi}e^{-\alpha r'} - \frac{\beta^2}{\alpha^2}\frac{\beta^3}{8\pi}e^{-\beta r'} + (\frac{\beta^2}{\alpha^2}-1)\delta(r')]\ . \end{aligned} \tag{12}$$

Here, α and β are chosen respectively to be $2.22\ fm^{-1}$ and $3.64\ fm^{-1}$, and $x = 0.594$, $\lambda = 0.145$, so that the root mean square radii of the proton and the neutron are $0.826\ fm$ and 0 respectively.

The magnetic moment of the nucleons and the mean square charge radius $\langle r_E^2 \rangle$ and magnetic moment radius $\langle r_M^2 \rangle$ can be defined as follows

$$(\vec{\mu})_Z = \frac{e}{2c}\int_V \rho_{ch}(r)(\vec{r}\times\vec{v})_Z\, d\tau\ ,$$
$$\langle r_E^2 \rangle = \int_V r^2 \rho_{ch}(r)\ d\tau\ ,$$
$$\langle r_M^2(proton) \rangle = \frac{1}{2.79\mu_N}\ \frac{e}{2c}\int_V r^2 \rho_{p,ch}(r)(\vec{r}\times\vec{v})_Z\, d\tau\ ,$$
$$\langle r_M^2(neutron) \rangle = \frac{1}{-1.911\mu_N}\ \frac{e}{2c}\int_V r^2 \rho_{n,ch}(r)(\vec{r}\times\vec{v})_Z\, d\tau\ , \qquad (13)$$

where μ_N is nuclear magneton.

From eqs.(8),(12) and (13), we can calculate magnetic moments and root mean square radii of charge and moment of the nucleons. Results are shows in Table 1.

We can also suppose that tachyonic pion mass distribution function $\rho_{mass}(r')$ (normalized to unit volume integral) in its rest frame as follows

$$\rho_{mass}(r') = \lambda\frac{\alpha^3}{8\pi}e^{-\alpha r'} + (1-\lambda)\frac{\beta^3}{8\pi}e^{-\beta r'}. \qquad (14)$$

Here, α, β and λ are chosen as before so that the root mean square radius of $\rho_{mass}(r')$ is $1.061\ fm$.

The expression for the z-component of mechanical angular momentum (spin) is

$$(\vec{S})_Z = m_1\int \rho_{mass}(r)(\vec{r}\times\vec{v})_Z\, d\tau\ . \qquad (15)$$

Using the following mass density and volume element superluminal transformations

$$\rho_{mass}(r) = \frac{\rho_{mass}(r')}{v^2-1}, \quad d\tau = d\tau' \ (v^2-1)^{\frac{1}{2}} , \tag{16}$$

we can calculate the spin of the nucleons. Results are also shown in Table 1.

	proton	neutron
spin	1/2 $\hbar$	1/2 $\hbar$
magnetic moment	2.79 μ_N	-1.911 μ_N
$\langle r_E^2 \rangle^{1/2}$	0.826 fm	0
$\langle r_M^2 \rangle^{1/2}$	0.916 fm	1.572 fm

Table 1. Spin, magnetic moment, rms radii of the charge and magnetic moments of the nucleons calculated by the model of the bradyon-tachyonic pion compound.

Ⅲ. Resonances

In inelastic electron-proton scattering, the scattered electron spectrum contains two features. First, one sees a number of bumps that correspond to excited states of the proton[10,14] and they are called resonances. Moreover, the radial dimensions of the excited states represented by the bumps are comparable to the dimensions of the proton itself in its unexcited condition[10].

The second feature is called continuum: inelastic scattering for continuum behaves nearly as if it were produced by point scatterers inside the proton. But the nature of these point scatterers and their relation to the observed or postulated particles is not yet clear[14].

So, from now on, we would like to explain these by our model of a bradyon and a tachyonic pion compounds. In analogy with bradyonic case we might assume the Lagrangian for a free tachyon[7]

$$L = m_1 \ (v^2-1)^{\frac{1}{2}}, \qquad (v^2 > 1) \ , \tag{17}$$

and in the usual way

$$\begin{aligned} p_i &= \frac{\partial L}{\partial v_i} \\ &= \frac{\partial}{\partial v_i} \{m_1(v^2-1)^{\frac{1}{2}}\} \\ &= \frac{m_1 v_i}{(v^2-1)^{\frac{1}{2}}} \ . \end{aligned} \tag{18}$$

Then the total energy E of the tachyon can be defined as

$$\begin{aligned} E &= p_i v_i - L \\ &= \frac{m_1}{(v^2-1)^{\frac{1}{2}}} \ . \end{aligned} \tag{19}$$

If the trapped tachyon is no longer free, we can write as usual the Lagrangian with the potential U as follows

$$L = m_1(v^2-1)^{\frac{1}{2}} - U \ , \tag{20}$$

where we supposed that the potential has the following form

$$U = \frac{Nm_0m_1}{r(v^2-1)^{\frac{1}{2}}} \quad . \tag{21}$$

Here we will consider only the uniform circular motion, that is, motion in which the magnitude of the velocity and rms radius r of the tachyonic pion is constant. Since we can put $\dot{r} = 0 = \ddot{\theta}$, the canonical monmentum corresponding to a coordinate of rotation θ is given by

$$\begin{aligned} p_\theta &= \frac{\partial L}{\partial \dot{\theta}} \\ &= \frac{\partial}{\partial \dot{\theta}}[m_1(r^2\ \dot{\theta}^2-1)^{\frac{1}{2}} - \frac{Nm_0m_1}{r(r^2\ \dot{\theta}^2-1)^{\frac{1}{2}}}] \\ &= \frac{v}{(v^2-1)^{\frac{1}{2}}}[m_1r+\frac{Nm_0m_1}{v^2-1}] \ , \end{aligned} \tag{22}$$

and the Hamiltonian is

$$\begin{aligned} H &= p_i q_i - L \\ &= p_r\dot{r} + p_\theta\dot{\theta} - L \\ &= \frac{1}{(v^2-1)^{\frac{1}{2}}}[m_1+\frac{Nm_0m_1}{r}(1+\frac{v^2}{v^2-1})] \ . \end{aligned} \tag{23}$$

Similarly to the Bohr angular momentum quantization condition, we would like to postulate the canonical momentum quantization condition as follows

$$p_\theta = \frac{v}{(v^2-1)^{\frac{1}{2}}}[m_1 r + \frac{Nm_0m_1}{v^2-1}]$$

$$= (l+\frac{1}{2})\hbar. \qquad (l = 0, 1, 2, \cdots) \qquad (24)$$

Now, we want to determine the parameters so that the first excited nucleon state is $\Delta(1232)$. If we substitute arbitrary strong charge gg' ($=0.22\hbar c$) into Nm_0m_1, and $r = \langle r^2_{mass}\rangle^{\frac{1}{2}} = 1.061\ fm$, and $m_1 = 139.567\ MeV$, then eqs.(22) and (23) can be rewritten as

$$H = E_{tach}(l)$$

$$= \frac{1}{(v_l^2-1)^{\frac{1}{2}}}[139.567+40.916(1+\frac{v_l^2}{(v_l^2-1)^{\frac{1}{2}}}]\ (MeV),$$

$$p_\theta = \frac{v_l}{(v_l^2-1)^{\frac{1}{2}}}[0.7504+\frac{0.22}{v_l^2-1}]$$

$$= (l+\frac{1}{2})\hbar, \qquad (25)$$

where v_l is the speed and $E_{tach}(l)$ is the energy of the tachyonic pion corresponding to the integer l.

Since the radial dimensions of the excited nucleon states are comparable to its unexcited one, we assume that the rms radii of the tachyonic pion mass density distribution are constant. Then the total energy $E_{tot}(l)$ can be put as

$$E_{tot}(l) = E_{tach}(l) + 939 \quad (MeV) . \tag{26}$$

Thus the $E_{tot}(l)$ will be compared to the experimental masses of N and Δ resonances [16], and these results are shown in Table 2 and Figure 1.

l	Theoretical		Experimental	
	$E_{tach}(l)$ (MeV)	$E_{tot}(l)$ (MeV)	particle	mass (MeV)
0	0	939	p , n	938, 939
1	295	1234	Δ(1232)	1230 ~ 1234
2	526	1465	N(1470)	1400 ~ 1480
3	739	1678	N(1680)	1670 ~ 1690
4	945	1884	Δ(2080)	1850 ~ 2000
5	1148	2087	N(2080)	1830 ~ 2100
6	1347	2286	Δ(2300)	2204 ~ 2450
7	1545	2484	Δ(2500)	2468(± 50)
8	1742	2681	N(2700)	2162 ~ 3000
9	1938	2877	Δ(2850)	2800 ~ 2900
10	2133	3072	N(~3000), Δ(~3000)	3100, 3200 (± 200)
11	2327	3266	Δ(~3000)	3300(± 200)
12	2521	3460	N(~3000)	3500 (± 200)
13	2714	3653	Δ(~3000)	3700 (± 200)
14	2907	3846	N(~3000)	3800 (± 200)
15	3099	4038	N(~3000)	4100 (± 200)

Table 2. Comparison of the theoretical and experimental masses of N and Δ resonances

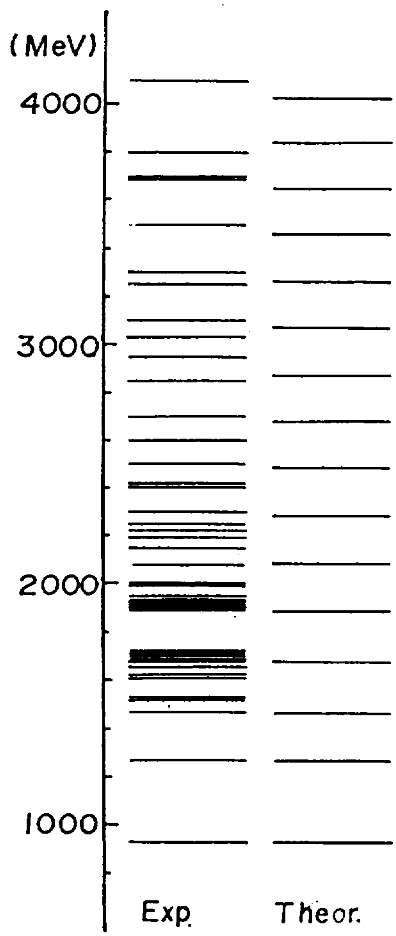

Figure 1. Ground state and excited states of the nucleon

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초 록

브라디온과 초광속 파이온으로 구성된 핵자의 구조가 이 논문에서 논의되었다. 정지 좌표계에서 초광속 파이온의 질량과 전하 밀도 분포가 지수함수로 나타내어 진다고 가정하면, 우리들은 핵자의 스핀, 자기능률 및 핵자의 전하와 자기능률 반지름들을 계산할 수 있다. 또한 초광속 파이온의 유한한 속도가 핵자의 들뜬 상태를 야기시킨다고 가정한다면, 그때 우리는 바른 운동량(canonical momentum) 양자화 조건을 가정함으로써 핵자와 델타 공명들의 질량들을 계산할 수 있다.