Design of 2- D Wave Digital Filters Having Circular Symmetry

Lee Yong-hak

Circular Symmetry 特性을 가진 2- D Wave Digital Filters의 設計

李 鎔 鶴

Summary

Some recent results for the design of 2- D wave digital filters with circular symmetry by making use of a reduced McClellan transformation are introduced. The rsulting filters have a very good circularly symmetric characteristic. The performances of filters with moderate degree can satisfy a certain practical requirements. The realization of the filter is also discussed.

INTRODUCTION

Two-dimensional (2- D) digital filter with circularly symmetric magnitude response are frequently needed for 2- D digital signal processing such as image processing (Huang, 1975). Hence the design of filters of this kind has been drawing the interest of many researchers. Various design approaches have appeared in the literature (Costa and Venetssanolos, 1974.) Filters designed by using the rotation and the spectral transformation require recursion of data in different direction and so they are not appropriate to the real time application (Huang, 1975). Various computer aided optimization approaches can produce transfer functions having nearly circularly symmetric property, however, involve difficulty to ensure the stability. Besides, the obtained transfer functions need not to have a passive network counterpart which is desirable from various consideration (Fettweis, 1972).

In (Rajan and Swamy, 1978), Rajan and Swamy proposed a straight forward method to design circularly symmetric low pass digital filters of arbitrary order. The resulting filter possesses good circular symmetry in the

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neighbourhood of the origin and controllable selectivity. Especially, the filter is guaranteed to be stable and causal, having rational transfer function with product separable denominator. However, the results of the method seem not so satisfactory in a few aspects. First, for the case of equiripple error approximation, the circularness in the transient band is not good. This is inevitable for the method, because equiripple approximation will disturb coefficients of the numerator of transfer function as to deflect the numerator from exponential approximation, which is required for circular symmetry. Second, the response in the stop band is monotonic (has no zeros). Consequently, it is difficult to produce a more sharp cut off characteristic. Third, once the denominator of the transfer function has been fixed by the exponential approximation, the characteristic will depend on the numerator. The method shows the best result can be obtained when the numerator degree is nearly half of the denominator degree. This implies a certain potential has not be utilized.

This paper propose an alternative method by making use of the McClellan transformation reduced by A. Fetweis for updating the design of numerator of transfer function. As a result, the obtained filters have all good features as ones in (Rajan and Swamy, 1978). Furthermore, very good circular symmetry in the transient band and more sharp cut off characteristic can be obtained. Equiripple can be achieved in pass band as well as in stop band, if required. The performances of filters with moderate degree can satisfy a certain practical requirement.

Transfer function obtained in the presented paper has a passive network counterpart and can be realized by wave digital filter (WDF) (Fettweis, 1971). This is practically very important for 2- D recursive filters. The resulting filter is then not only automatically stable under linear condition, but all small and large scale parasitic oscillations can be fully suppressed. Forced-response stability can be guaranteed, shorter coefficient wordlength can be adopted for a suitable dynamic range. All the situations are the same as in 1- D case (Fettweis, 1986).

As a contrast, in (Rajan and Swamy, 1978) the authers provide a transfer function in z-variable form as a final result, which will correspond to a conventional 2- D digital filter and, in view of the resulting performance, be usually less attractive.

Linear phase is shown to be very important in 2- D filtering applications (Huang, Burnett and Deczky, 1975). Recursive filters, however, usually have nonlinear phase characteristic. The correction is therefore required in usual case. For the filters obtained in this paper the phase correction is relatively easy.

THE DESIGN APPROACH

In order to produce a transfer function that is suitable to be realized by a 2- D WDF, we will adopt the equivalent complex Frequency φ defined by

$$\psi = \frac{z-1}{z+1} \qquad \tanh(pT/2) \tag{1}$$

$$z = e^{PT}, \qquad F = 1/T \tag{2a,b}$$

where z is the usual z-variable and F the sampling rate, p the usual complex Frequency. Let $\psi = j\phi$ abd $p = j\bar{\omega}$, we have

$$\varphi = \tan(\omega/2) \tag{3}$$

where $\omega = \bar{\omega}T$, the normalized real frequency.

In (Rajan and Swamy, 1978) it has been shown by Rajan and Swamy that a two variable rational function possessing quadrantal symmetry is causal and stable if and only if its denominator is expressible as a product of two one-variable Hurwitz polynomials. Thus, the desired magnitude squared function of the analog transfer function should have the form as

$$|H(j\varphi_1, j\varphi_2)|^2 = \frac{F^2(\varphi_1^2 \cdot \varphi_2^2)}{G(\varphi_1^2 \cdot G(\varphi_2^2)} \quad (4)$$

where G(φ^2) can be chosen in the same way as in (Rajan and Swamy, 1978). That is, we should have

$$G(\varphi^2) = \hat{G}(\omega^2) \approx e^{\alpha'\omega'} = e^{\alpha'(2\arctan\varphi)^2}$$

In the range $|\varphi|\langle 1(\omega\langle\pi/2)$, arctan $\omega\approx\varphi$ Let $G(\varphi^2)\approx e^{\alpha\varphi^2}$, we have

$$G(\varphi^2) = \sum_{i=0}^{n} \frac{(\alpha\varphi^2)^i}{i!} \quad (5)$$

where n and α can be determined in an appropriate way. Consequently, the contribution of the denominator in eq. (4) to the magnitude response is circularly symmetric in the neighbourhood of the origin.

Since the denominator is almost fixed, the choice of the numerator will be the key for obtaining a desired characteristic. Our procedure consists of two steps. First step, we search for a one-variable function F(φ^2) so that $F^2(\varphi^2)/G(\varphi^2)$ possesses a desired 1-D magnitude squared response (MSR). This can be done by first determining a approximate guess for $F^2(\varphi^2)$ using the simple interpolation and then by optimization using Remez exchange algorithm (Blum, 1972). In this case, 1/G(φ^2) can be regarded as a weight. Once $F^2(\varphi^2)/G(\varphi^2)$ has been determined, the second step is McClellan transformation for *only* the numerator. In (Fetweis, 1977), A. Fettweis proposed a reduced form as follows (for 2- D case)

$$\varphi^2 = \varphi_1^2 + \varphi_2^2 + k\varphi_1^2 \cdot \varphi_2^2 \quad (6)$$

When k= 1~2 (typically, k= 1.33), the contour diagrams of eq. (6) in ω-plane exhibit very good circular symmetry even in some distance from the origin. Additionally, for different distance of transient band from the origin, k can be chosen so that the best circular symmetry can be achieved. Thus, the obtained 2- D MSR has the form

$$|H(j\varphi_1, j\varphi_2)|^2 = \frac{F^2(\varphi_1^2+\varphi_2^2+k\varphi_1^2\cdot\varphi_2^2)}{G(\varphi_1^2)G(\varphi_2^2)} \quad (7)$$

It should be mentioned that the results in (8) can be viewed of a special case of eq. (7) when k= 0.

As it can be illustrated that the 2- D MSR in (7) will lead a realizable 2- D transfer function by using WDF.

AN ILLUSTRATED EXAMPLE

Given the specifications of a 2- D low-pass digital filter as follows: Ripple in pass band

$$(0 \leqq \sqrt{\omega_1^2+\omega_2^2} \leqq 0.3) : \geqq .1 \text{ dB},$$

$$(\sqrt{\omega_1^2+\omega_2^2} > 1.35) : > 40 \text{ dB}$$

Attenuation in stop band $(\sqrt{\omega_1^2+\omega_2^2} > 1.35)$:40dB. In order to produce a suitable F(φ^2), the following observation would be beneficial. Under the McClellan transformation, a real zero of F(φ^2) will form a zero circle of MSR. A imaginary zero of F(φ^2) will contribute a concave top to MSR. A comolex zero of F(φ^2), if the location is suitable, may introduce a concave ripple to MSR. For the given specifications, it is successful to choose the degree of $F(\varphi^2)$ to be 6. Thus, let

$$F(\varphi^2) = 1 + a_1\varphi^2 + a_2\varphi^4 + a_3\varphi^6 \qquad (8)$$

The degree of $G(\varphi^2)$ should be higher than 2×6, we use degree of $G(\varphi^2)$ 2×8. By carrying out the procedure introduced previously, a 2-D MSR can be found in the form of eq. (7) with

$$G(\varphi^2) = \sum_{i=0}^{8} \frac{(a_0\varphi^2)^i}{i!} \qquad (9)$$

$$F(\varphi^2) = \lambda(\varphi^2-\alpha_1)(\varphi^2-\alpha_2)(\varphi^2-\alpha_3) \qquad (10)$$

where $\alpha_0 = 13.635$, $\alpha_1 = 0.6601$, $\alpha_2 = -0.0993$, $\alpha_3 = 1.419$, $\lambda = 10.68943$

Consider the McClellan transformation of a factor $F_i(\varphi^2) = \varphi^2 - \alpha_i$. Thus

$$F_i(\varphi_1^2, \varphi_2^2) = k\varphi_1^2 \cdot \varphi_2^2 + \varphi_1^2 + \varphi_2^2 - \alpha_i$$

$$= (\beta_i\varphi_1^2 + \gamma_i)(\varphi_2^2 + \sigma_i) + (\beta_i\varphi_2^2 + \gamma_i)(\varphi_1^2 + \sigma_i) \qquad (11)$$

where

$$\beta_i = k/2 \qquad (12a)$$

$$\gamma_i = (1 + \sqrt{1+\alpha_i k})/12 \qquad (12b)$$

$$\sigma_i = -\alpha_i/\lambda_i \qquad (12c)$$

As it can be seen that the expression of (11) is favorable for realization by WDF. And, this is feasible only when $1+\alpha_i k \geqq 0$. In usual case, this conditon is no problem. Since $G(\varphi^2)$ is a even function in φ, $G(-\psi^2)$ can be factorized in the form $G(-\psi^2) = g(\psi)\cdot g(-\psi)$, where $g(\psi)$ is a Hurwitz polynomial. After factorization of $G(\varphi^2)$, the following transfer function can be formed

$$H(\psi_1, \psi_2) = \frac{\lambda' \prod_{i=1}^{3} ((\psi_1^2 - \gamma_i')(\psi_2^2 - \sigma_i) + (\psi_2^2 - \gamma_i')(\psi_1^2 - \sigma_i))}{g(\psi_1)g(\psi_2)} \qquad (13)$$

where $\gamma_i' = \gamma_i/\beta_i$, and

$$g(\psi) = \prod_{i=1}^{4} g_i(\psi) = \prod_{i=1}^{4} (\psi^2 + a_i\psi + b_i) \qquad (14)$$

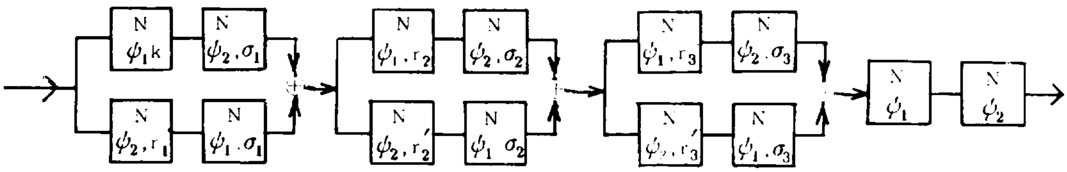
It is convenient for realization to give the freedom of choice. The transfer function (13) has a realization as shown in Figure 1. Note that every block of the filter corresponds to a passive network: if the numerator is quadratic, then the block corresponds to a Brune or typ-C section, depending on the sign of the constant term; if the numerator is a constant, then the block correponds to a ladder network. In any case, a realization by WDF is always possible (Fettweis, 1986). Thus, the filter can be implemented by using 14 2-order 1-D WDF's.

In Figure 2, a perspective view of $|H(j\omega_1, j\omega_2)|$ in eq. (13) is given, where

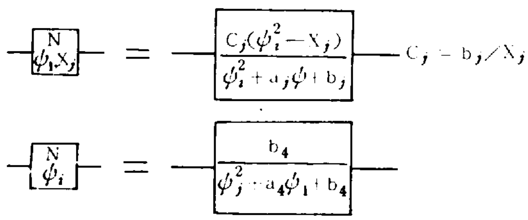
$$\hat{H}(j\omega_1, j\omega_2) = H(j\varphi_1, j\varphi_2)|\varphi_i = \tan(\omega_i/2),\ i = 1, 2 \qquad (15)$$

The contour plots of $|\hat{H}(j\omega_1, j\omega_2)|$ is shown in Figure 3. As it can be seen that very good circular symmetry has been achieved. The obtained ripple in the range $0 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 0.3$ is 0.05 dB and the attenuation in stop band $\sqrt{\omega_1^2 + \omega_2^2} > 1.35$ is 39.65 dB.

If a steeper transient band is requested, we should introduced some complex zeros in $F(\varphi^2)$. In this case $F(\varphi^2)$ can not be put into the form of eq. (10). However, at any rate, $F(\varphi^2)$ can be designed to be a product of 4-order polynomial. Under the McClellan transformation, the resulting transfer function is also realizable by combination of Darlinton typ- D sections.



(a) Structure of the filter



(b) Building blocks of the filter

Fig. 1. A realization of the transfer function

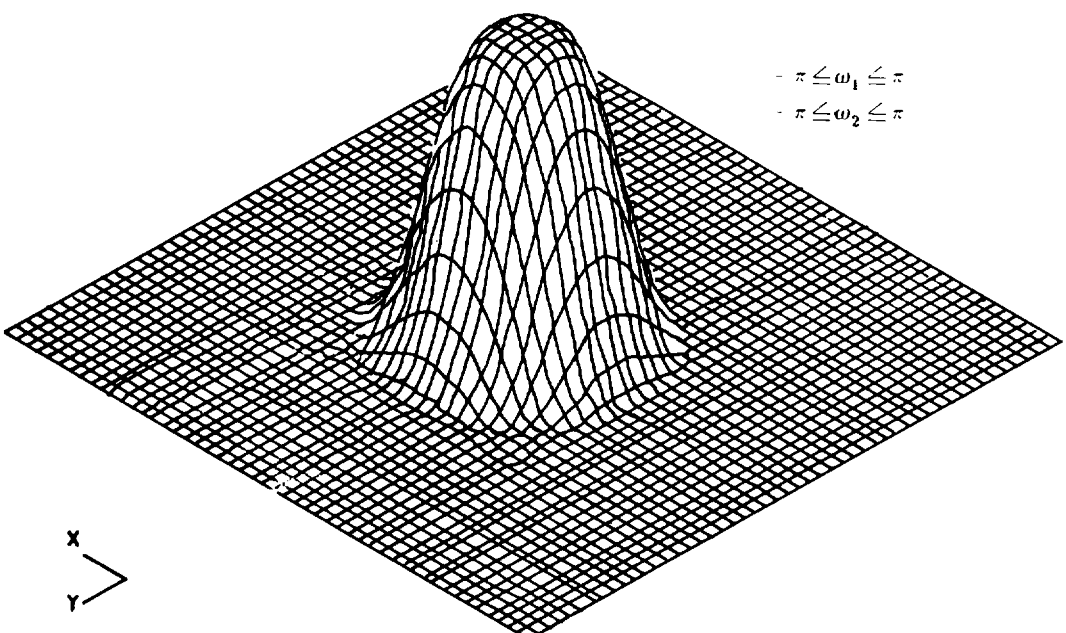
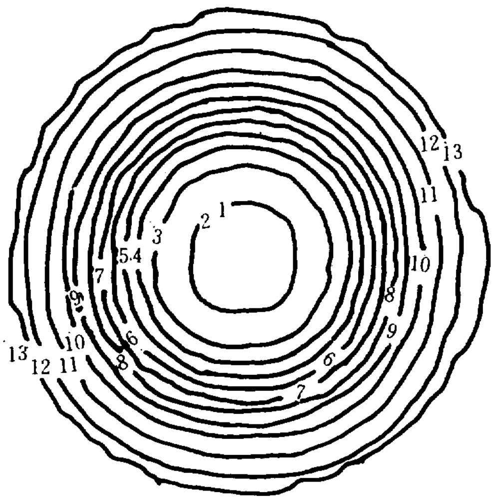


Fig. 2. The perspective view of magnitude response in (13)



CONTOUR KEY	
1	0.990
2	0.989
3	0.900
4	0.800
5	0.700
6	0.600
7	0.500
8	0.400
9	0.300
10	0.200
11	0.100
12	0.032
13	0.010

$-\pi \leq \omega_1 \leq \pi$

$-\pi \leq \omega_2 \leq \pi$

Fig. 3. The contour plot of magnitude response in (13)

CONCLUSIONS

This paper has introduced results for the design of 2- D WDF with circular symmetry by making use of a reduced McClellan transformation. The obtained filters have a very good circularly symmetric characteristic. The realization of the filters is also discussed.

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國 文 抄 錄

本 論文에서는 McClellan 變換法을 利用하여 Circular Symmetry 特性을 가진 2- D Wave Digital Filters를 設計하였다.

設計된 Filters는 매우 좋은 Circularly Symmetric 特性을 가졌으며, Filters의 實現에 대해서도 論하였다.