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Robust H_∞ State Feedback Controller Design of LPD System with the Time Varying State Delay

Jin-Shig Kang* and Boo-Sik Kang**

상태벡터에 시변 시간지연을 포함하는 LPD 시스템의 강인한 H_∞ 상태 되먹임 제어기 설계

강 진 식* · 강 부 식**

Abstract

In this paper, we considered the LPD system with the time varying state delay. Quadratic stability and γ -performance conditions are derived in terms of linear matrix inequalities(LMI's). The well known H_∞ state feedback controller synthesis problem for state delayed LPD system is solved and the equation of computing the feedback gain are presented. The closed loop quadratic stability and γ -performance conditions are derived and the equivalent stability and γ -performance conditions are characterized by linear matrix inequalities(LMI's). Some comments on the computational complexity of LPD system are mentioned.

Keywords : LPD system, State delay, Quadratic stability, γ -performance, LMI

I. Introduction

In this paper we considered a finite dimensional linear parameter dependent (LPD) system with state delay whose entries of state space representation depends on the piecewise continuous time varying parameter vector.

Control of LPD system is mainly treated in the gain scheduling methodology. For this method, the robust stability and robust performance condition is derived by Shamma[1]. By B.G. Scott[2], the quadratic

* 제주대학교 통신공학과
Dept. of Telecommunication Eng., Cheju Nat'l Univ.
** 제주 관광전문대학 국제정보통신과
Dept. of International Telecommunication, Cheju Tourism College.

stability and γ-performance condition is derived in the forms of LMI and controller synthesis is considered. W. Fen[3] present the LQG and L_2 design of same type of system.

Time delayed systems are treated by some authors and useful results are reported.[1-8] J.H. Lee, *et al.*[4] treats stable delay system and suggest memoryless H_∞ controller. J.S. Lou, *et al*[5] is studied delay dependent robust stability and M.S. Mahmoold and N.F. Al-Muthari[6] present the design of robust controller for delayed system.

Most of the mechanical system and the chemical reactor may contains time delay and the dynamic equations of which are depend on the time-varying parameter vector.

In this paper, state delayed LPD system is considered. The parameter vector is assumed to be piecewise continuous and real-time measurable. In section II, the parameter variation set, plant dynamics and general statements on H_∞ controller design are reviewed. The quadratic stability and γ -performance problem is analysed in section III. In section IV, the state feedback design is treated, the closed-loop stability and γ -performance conditions are characterized in terms on LMI and some comments on computational complexity of LMI's of the LPD system design are mentioned.

II. Dynamics and H_∞ design problems of state delayed LPD system

In this section, dynamics of the state delayed LPD systems are considered and general H_∞ design problem is reviewed. In order to describe the state delayed system, we firstly define the parameter variation set.

2.1. Parameter variation set

Most of the mechanical system, chemical reactor and physical stems can be modelled by LPD system whose dynamics are depend on the time varying parameter vectors and in which the time delay terms are included many of the case. On the parameter variation set, it is assumed that we have real time information about a vector valued parameter whose value is associated with the dynamics of the plant in some known way. The following definition is characterize the parameter variation set.

Definition 2.1.[2] Given a compact subset $P \in R^s$, the parameter variation set F_P denotes the set of all piecewise continuous functions mapping R^+(time) into P with a finite number of discontinuities in any interval. For each ρ_i and all t, there exist $\rho_{min}(>0)$ and $\rho_{max}(>0)$ such that $\rho_{min} \le |\rho_i(t)| \le \rho_{max}$ where α_i, β_i is the minimum value and maximum value of ρ_i.

By the definition 2.1, the controller which is designed for each ρ_i can be scheduled and for scheduled controller, the stability and performance conditions are preserved.

2.2. Dynamics of state delayed LPD system

The dynamic equations of the LPD system with delayed state is described by

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(\rho) & A_d(\rho) & B_1(\rho) & B_2(\rho) \\ C_1(\rho) & C_{1d}(\rho) & D_{11}(\rho) & D_{12}(\rho) \\ C_2(\rho) & C_{2d}(\rho) & D_{21}(\rho) & D_{22}(\rho) \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d_\tau(t)) \\ w(t) \\ u(t) \end{bmatrix} \quad (1)$$

where $\rho \in F_P$, $\dot{x}, x, x_d \in R_n$, $w \in R^{n_w}$, $z \in R^{n_z}$, $u \in R^{n_u}$, and $y \in R^{n_y}$. Without loss of generality, it is assumed that the following conditions on the dynamic equations are hold for all $\rho \in P$.

A1) $D_{12}(\rho(t))$ is full column rank.

A2) $D_{21}(\rho(t))$ is full row rank.

A3) $D_{22} = \mathbf{0}$.

A4) $D_{12} = [0,\ I_{n_u}]^T$ and $D_{21} = [0,\ I_{n_y}]$.

By norm preserving coordinate transform, the generality of assumption A4) is preserved[2]. For notational purpose, we do not notate parameter value ρ in dynamic equations and matrix equations. Thus all variables and elements of matrices in this paper are parameter dependent value or matrix. Let us define the delayed state vector as $x_d^\tau(t) := x(t-d_\tau(t))$. By these assumption, the dynamic equation is

$$\begin{bmatrix} \dot{x}(t) \\ z_1(t) \\ z_2(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A & A_d & B_{11} & B_{12} & B_2 \\ C_{11} & C_{11d} & D_{1111} & D_{1112} & \mathbf{0} \\ C_{12} & C_{12d} & D_{1121} & D_{1122} & I_{z_2} \\ C_2 & C_{2d} & \mathbf{0} & I_{w2} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x(t) \\ x_d^\tau(t) \\ w_1(t) \\ w_2(t) \\ u(t) \end{bmatrix} \quad (2)$$

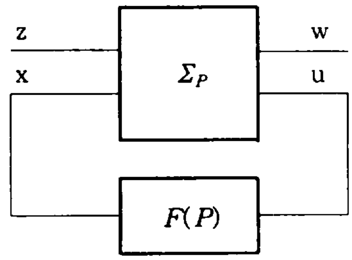

Fig. 1 The closed loop structure

The equation (2) is used for the analysis of the closed loop stability and performance in section 4.

2.3. H_∞ Design Problem

The closed loop structure of general H_∞ design is shown in Fig.1. Then the design problem is "Find a state feedback controller $F(P)$ such that for all $\rho(t) \in F_P$ (for all t), Stabilizes Σ_P and Minimize $\| T_{zw} \|_\infty$." For LTI system[2], this H_∞ problem is equivalent to the L_2 problem, i.e., minimization of $\| T_{zw} \|_\infty$ is equivalent to the minimization of $\| T_{zw} \|_2$. In this paper, the controller is designed for each ρ_i. This means that the L_2 design is equivalent to H_∞ design.

III. Stability and performance of state delayed LPD system.

In this section, we characterize the stability and γ-performance of the state delayed LPD system in terms of LMI's.

3.1. Open loop stability and γ performance

The well known Lyapunov method is used for stability analysis.. The following lemma states the stability of the state delayed LPD system.

Lemma 3.1: Given state delayed LPD system Σ_P, if there exists $P=P^T>0$ and $S=S^T>0$ such that

$$\begin{bmatrix} A^TP+PA & PA_d \\ A_d^TP & -(1-\dot{d}_\tau(t))S \end{bmatrix} < 0 \qquad (3)$$

then the given system is quadratically stable over F_P.

proof) Let us select a Lyapunov functional $V(x,\ x_d)$ as

$$V(x,\ x_d)=\ x^T(t)Px(t)+\int_{t-d_\tau}^{t} x^T(\tau)Sx(\tau)d\tau \qquad (4)$$

Taking the time derivative, we can obtain

$$\frac{d}{dt}V(x(t),\ x(t-d_\tau(t))=$$

$$\dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t)+x^T(t)Sx(t)$$

$$-(1-\dot{d}_\tau(t))\ x_d^{T}(t)S\,x_d(t) \qquad (5)$$

For stability analysis, we assume the zero input. By substitute of equation(1) into equation(5) and some algebraic manipulation, equation(5) is

$$\dot{V}(x,\ x_d)=[x^T\ x_d^T]\cdot$$

$$\begin{bmatrix} A^TP+PA+S & PA_d \\ A_d^TP & -(1-\dot{d}_\tau)S \end{bmatrix}\begin{bmatrix} x \\ x_d \end{bmatrix} \qquad (6)$$

Thus $V(x,\ x_d)<0$ if and only if there exists $P(=P^T>0)$ and $S(=S^T>0)$ such that

$$\begin{bmatrix} A^TP+PA+S & PA_d \\ A_d^TP & -(1-\dot{d}_\tau)S \end{bmatrix}<0 \qquad (7)$$

if time delay is not function of time, then equation(7) is abbreviated by

$$\begin{bmatrix} A^TP+PA+S & PA_d \\ A_d^TP & -S \end{bmatrix}<0 \qquad (8)$$

The equation(7) and (8) characterize the open loop stability for the given system. The following lemma characterize the γ-performance condition.

Lemma 3.2: Given P, $\gamma>0$ and state delayed LPD system, the quadratic γ-performance problem is solvable if there exist $P=P^T>0$ and $S=S^T>0$ such that

$$\begin{bmatrix} A^TP+PA+S & PA_d & PB & C_1^T \\ A_d^TP & -(1-\dot{d}_\tau)S & 0 & C_{1d} \\ B^T & 0 & -\gamma^2 I & D_{11} \\ C_1 & C_{1d} & D_{11} & -I \end{bmatrix}<0 \qquad (9)$$

proof) Let us select the performance measure J as

$$J=\int_0^\infty [z^T(t)\ z(t)-\gamma^2 w^T(t)w(t)]dt \qquad (10)$$

Now, we assume the zero initial conditions and for non zero $w(t)\in L_2[0,\ \infty)$, the following is hold

$$J=\ \int_0^\infty [z^T(t)\ z(t)-\gamma^2 w^T(t)w(t)+\dot{V}(\cdot)]dt$$

$$-V(\infty) \tag{11}$$

since $V(0)=0$, thus

$$J \le \int_0^\infty [z^T(t)\ z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(\cdot)]dt \tag{12}$$

and substitution and let $\zeta=[x,\ x_d,\ w]^T$ then

$$J \le \int_0^\infty [\zeta^T(t)Q\zeta(t)]dt \tag{13}$$

where Q is obtained by equation(14) and which is Schur complement of $\hat{Q}$ in equation(15).

$$Q=\begin{bmatrix} A^TP+PA+S+C_1^TC_1 & PA_d+C_1^TC_{1d} & PB+C_1^TD_{11} \\ A_d^TP+C_{1d}^TC_1 & -(1-\dot{d}_r)S+C_{1d}^TC_{1d} & C_{1d}^TD_{11} \\ B^TP+D_{11}^TC_1 & D_{11}^TC_{1d} & -\gamma^2 I+D_{11}^TD_{11} \end{bmatrix} \tag{14}$$

$$\hat{Q}=\begin{bmatrix} A^TP+PA+S & PA_d & PB_1 & C_1^T \\ A_d^TP & -(1-\dot{d}_r)S & 0 & C_{1d}^T \\ B_1^TP & 0 & -\gamma^2 I & D_{11}^T \\ C_1 & C_{1d} & D_{11} & I \end{bmatrix} \tag{15}$$

3.2. Closed loop γ-performance

Let us denote the controller dynamics as

$$\begin{bmatrix} \dot{x}_K(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \begin{bmatrix} x_K(t) \\ y(t) \end{bmatrix} \tag{16}$$

Let $x_{cl}(t) := [x^T(t)\ x_K^T(t)]^T$ then the closed loop dynamic equation is

$$\begin{bmatrix} \dot{x}_c(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A_c & A_{cd} & B_c \\ C_c & C_{cd} & D_c \end{bmatrix} \begin{bmatrix} x_c(t) \\ x_{cd}(t) \\ w(t) \end{bmatrix} \tag{17}$$

where

$$A_c=\begin{bmatrix} A+B_2D_K(\rho i)C_2 & B_2C_K \\ B_KC_2 & A_K \end{bmatrix},$$

$$A_{cd}=\begin{bmatrix} A_d+B_2D_KC_{2d} \\ B_KC_{2d} \end{bmatrix},$$

$$B_c=\begin{bmatrix} B_1+B_2D_KD_{21} \\ B_KD_{21} \end{bmatrix},$$

$$C_c=[C_1+D_{12}D_KC_2\ \ D_{12}C_K],$$

$$C_{cd}=[C_{1d}+D_{12}D_kC_{2d}],$$

$$D_c=D_{11}+D_{12}D_KD_{21}$$

The following lemma states the closed loop γ-performance.

Lemma 3.2 : Given LPD system with state delay, the quadratic γ -performance problem is solvable if there exists an $m>0$, $X\in R^{(n+m)\times(n+m)}$, $X=X^T>0$, $S_c=S_c^T>0$ and a continuous and bounded matrix function $K:R^s \rightarrow R^{(n_u+m)\times(n_y+m)}$ such that, for all $\rho \in P$

$$\begin{bmatrix} A_c^TX+XA_c+S & XA_{cd} & XB_c & C_c^T \\ A_{cd}^TX & -(1-\dot{d}_r)S_{11} & 0 & C_{cld} \\ B_c^TX & 0 & -\gamma^2 I & D_c^T \\ C_c & C_{cld} & D_c & -I \end{bmatrix} < 0 \tag{18}$$

Here, S_{11} is positive definite and defined as

$$S=\begin{bmatrix} S_{11} & 0 \\ 0 & 0 \end{bmatrix}. \tag{19}$$

proof) The same procedure of lemma

3.1.

IV. State feedback controller design

In this section, the state feedback problem is considered and the equivalent conditions of equation(18) is derived. The computational complexity, occurred in the design of LPD system, is stated in the last sub-section.

4.1. Stabilization by state feedback

Consider the following state delayed LPD system

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A & A_d & B_1 & B_2 \\ C_1 & C_{1d} & D_{11} & D_{12} \\ I_n & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x(t) \\ x_d(t) \\ w(t) \\ u(t) \end{bmatrix} \qquad (20)$$

The solvability of quadratic stabilizability by state feedback is characterized by the following theorem.

Theorem 4.1. : The quadratic state feedback problem for the LPD system with state delay is solvable if and only if there exist a continuous function $F:R^s \to R^{n_u \times n}$, a matrix $Z \in R^{n \times n}$, $Z = Z^T > 0$ and a scalar $\gamma > 0$ such that

$$\begin{bmatrix} A_F^T Z + PA_F & PA_d & PB_1 & C_F^T \\ A_d^T P & -(1-\beta)S_{11} & \mathbf{0} & C_{1d}^T \\ B_1^T & \mathbf{0} & -\gamma^2 I & D_{11}^T \\ C_F & C_{1d} & D_{11} & -I \end{bmatrix} < 0 \qquad (21)$$

where

$A_F = A + B_2 F$, $C_F = C_1 + D_{12} F$

proof) Same procedure with lemme 3.1 and 3.2.

Theorem 4.2. : The equation(21) in the theorem 4.1 is equivalent to the equation(22).

$$\begin{bmatrix} Z\bar{A}^T + \bar{A}Z + ZSZ + B_2^T B_2 & \bar{A}_d & \bar{B}_1 \\ \bar{A}_d^T & -(1-\beta)S & \mathbf{0} \\ \bar{B}_1^T & \mathbf{0} & -\gamma^2 I \end{bmatrix} < 0 \qquad (22)$$

where

$\bar{A} = A - B_2 C_1$, $\bar{A}_d = A_d - B_2 C_{1d}$,

$\bar{B}_1 = B_1 - B_2 D_{11}$

and the stabilizing state feedback gain is

$$F = -\gamma^{-2}(D_s^T D_s)^{-1}(B_s^T Y^{-1} + D_s^T C_s). \qquad (23)$$

proof) see appendix

4.2. Comments on the computational complexity

When we considering the LPD system, the computational problem is occurred. One way to avoid this problem is that grid-ing the parameter value, design controllers for each grid and then schedule the controllers. But this algorithm have constraints: one is the relationship between the grid interval and the stability condition, i.e., as grid interval is increase then computational complexity is decrease but stability margin is decrease. This problem may be weaken by the H_∞ loop shaping design. The H_∞ loop shaping design procedure is that selection of input output weighting functions and then design a controller for weighted plant. The design procedure by H_∞ loop shaping design is summarized as follows.

step1 : select the weighting functions as possible as insensitive with parameter variation.

step2 : grid-ing parameter values

step3 : compute the open loop gain for each grid and grouping the plants.

step4 : design controllers for dominant plants in each group.

step5 : scheduling the controllers.

By this algorithm, the number of controller can be reduced and computational complexity may be weakened.

V. Conclusion

The H_∞ design of state feedback controller for state delayed LPD system is considered. The design problems can be characterized by LMI's and state feedback gain is computed. The input delayed and both input and state delayed system is to be studied and the output feedback controller design is also treated in the near future.

VI. 요 약

본 논문에서는 상태변수에 시변 시간지연을 포함하는 LPD 시스템에 대하여 고찰하였다. quadratic 안정도 및 γ-성능조건을 선형행렬 부등식(LMI) 형태로 유도하였다. 주어진 시스템에 대한 H_∞ 상태 되먹임 제어문제에 대한 해를 제시하였으며 상태 되먹임 이득을 구하였다. 폐루프 quadratic 안정도및 γ-성능조건을 유도하였으며 등가조건을 LMI 형태로 구하였다. LPD 시스템 설계에서 계산상의 문제점을 고찰하였다.

Appendix

Proof of the theorem 4.1

=> Define R, U and $\hat{F}$ as

$$R=\begin{bmatrix} A^TP+PA+S & PA_d & PB_1 & C_1^T \\ A_d^TP & -(1-\dot{d}_r)S & 0 & C_{1d}^T \\ B_1^TP & 0 & -\gamma^2 I & D_{11}^T \\ C_1 & C_{1d} & D_{11} & -I \end{bmatrix} < 0 \tag{A1}$$

Let us select U and F as

$$U=\begin{bmatrix} PB_2 \\ 0 \\ 0 \\ I \end{bmatrix}, \quad \overline{F}=[F\ 0\ 0\ 0]$$

Then the equation(A1) can be rewrite as follows

$$R+U\overline{F}+\overline{F}^TU^T<0 \tag{A2}$$

Let $U_\perp$ is the orthogonal complement of U, then

$$U_\perp=\begin{bmatrix} Z & 0 & 0 \\ 0 & I_n & 0 \\ 0 & 0 & I \\ -B_1^T & 0 & 0 \end{bmatrix} \tag{A3}$$

where $Z=P^{-1}$. Pre-multiplication of $U_\perp^T$ and post multiplication of $U_\perp$ to the equation(A2) is

$$U_\perp^T R U < 0 \tag{A4}$$

and some simple algebraic manipulation, we can obtain the equation (22)

$\Leftarrow$ For sufficiency we show that the equation(22) establishes the existence of a matrix function F such that the equation (22) holds for all $\rho \in P$. The equation(22) can be written by

$$Y\tilde{A}^T + \tilde{A}Y + YQY + \tilde{R} < 0 \qquad (A5)$$

where

$$\tilde{A} = A_F + B_1 D_{11}^T(\gamma^2 I - D_{11}^T D_{11})^{-1} C_F + A_d + B_1 D_{11}^T(\gamma^2 I - D_{11}^T D_{11})^{-1} C_{1d}$$

$$Q = \gamma^2 C_F^T(\gamma^2 I - D_{11}^T D_{11})^{-1} C_{1d} + \gamma^2 C_{1d}^T[\gamma^2(\gamma^2 I - D_{11}^T D_{11})^{-1} C_F + \gamma^2 C_{1d}^T(\gamma^2 I - D_{11}^T D_{11})^{-1}]C_{1d} - (1 - \dot{d}_\tau)S$$

$$\tilde{R} = B_1(\gamma^2 I - D_{11}^T D_{11})^{-1} B_1^T$$

Let $Z = Y^{-1}$ and further expending of equation(A5) by using A_F, C_F, we can obtain

$$\left.\begin{matrix} A_s^T Z + ZA_s + ZB_1(\gamma^2 I - D_{11}^T D_{11})^{-1} B_1^T Z \\ -(1 - \dot{d}_\tau)S \end{matrix}\right\} ①$$

$$\left.\begin{matrix} +\gamma^2 F^T[B_s^T Z + D_s^T C_s] + \gamma^2[ZB_s + C_s^T D_s]F \\ F^T D_s^T D_s F + C_s^T C_s < 0 \end{matrix}\right\} ②$$

where

$$A_s = A + A_d + B_1 D_{11}(\gamma^2 I - D_{11}^T D_{11})^{-1} D_{11}^T(C_1 + C_{1d})$$

$$B_s = B_2 + B_1(\gamma^2 I - D_{11}^T D_{11})^{-1} D_{11}^T D_{12}$$

$$C_s = (\gamma^2 I - D_{11}^T D_{11})^{-\frac{1}{2}}(C_1 + C_{1d})$$

$$D_s = (\gamma^2 I - D_{11}^T D_{12})^{-\frac{1}{2}} D_{12}$$

The equation above can be divided into two parts one is matrix Riccatti inequality(①) and the other is the algebraic equation which containing state feedback gain(②). We note here that matrix Riccatti inequality is equivalent to the (1, 1) block Schur complement of the equation (22). Thus, we can obtain the state feedback gain F as

$$F = -\gamma^{-2}(D_s^T D_s)^{-1}(B_s^T Y^{-1} + D_s^T C_s) \qquad (A6)$$

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