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The Onset of Convective Instability of Viscoelastic Fluids in Porous Media

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다공성 매질층에 포화된 점탄성 유체의 대류 불안정 발생

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ABSTRACT

A theoretical analysis of thermal instability driven by buoyancy forces is conducted in an initially quiescent, horizontal porous layer saturated by viscoelastic fluids. Modified Darcy's law is used to explain characteristics of fluid motion. The linear stability theory is employed to find the critical condition of the onset of convective motion. The results of the linear stability analys shows that the overstabilty is prefer mode for a certain parameter range.

Key words : Convective instability, Viscoelastic fluid, Porous media

I. INTRODUCTION

From the beginning of this century the convective motion driven by buoyancy forces have attracted many researchers' interests. In this connection buoyancy-driven pheonomena in porous media are actively under investigation. It is well known that the buoyancy-driven phenomena in porous media have a wide variety of engineering applications, such as geothermal reservoirs, agricultural product storage system, packed-bed catalytic reactors, the pollutant transport in underground and the heat removal of nuclear power plants.

With Newtonian fluid system of slow heating Horton and Rogers[1] and Lapwood[2] conducted the theoretical analysis on the critical condition of the onset of buoyancy-driven motion in fluid-saturated horizontal porous layers. They employed Darcy's law to express the flow characteristics in porous layers and analysed the effect of Darcy number on the onset condition of buoyancy-

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driven convection. Katto and Masuoka[3] showed experimentally the effect of the Darcy number on the critical condition of the onset of buoyancy- driven convection. In case of Newtonian fluid, the critical condition to mark convective motion has been analyzed under the principle of the exchange of stabilities. But viscoelastic fluids like polymeric liquids can exhibit markedly different stability properties. For the Rayleigh-Benard problem, Vest and Arpaci[4], and Kolka and Ierley[5] analyzed overstability of Maxwell fluid and Oldroyd-B fluid, respectively. They confirm that the buoyancy forces can induce the periodic instability before the exchange of stabilities. Recently, Lee et al.[6] extended overstability of Benard-Marangoni problem into the viscoelastic fluids.

In the present study we investigate the linear stability on a horizontal porous layer which is staturated by viscoelastic fluid. Here will be shown that for a certain parameter range the periodic motion caused by overstability is replaced by stationary modes.

II. LINEAR STABILITY ANALYSIS

2.1 Governing Equations

The system considered here is an initially quiescent, fluid-saturated, horizontal porous layer of depth d, as shown in Fig. 1. The porous medium is homogeneous and isotropic, and saturated by viscoelastic fluid. The porous layer is heated slowly from below. For this system the governing equations of flow and temperature fields are expressed employing the Boussinesq approximation and modified Darcy's model[7]:

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\frac{\mu}{K}\left(\varepsilon_s \frac{\partial}{\partial t} + 1\right)\mathbf{u} = \left(\lambda_s \frac{\partial}{\partial t} + 1\right)(-\nabla P + \rho\, \mathbf{g}) \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) T = \alpha \nabla^2 T \tag{3}$$

$$\rho = \rho_0[1 - \beta(T - T_0)] \tag{4}$$

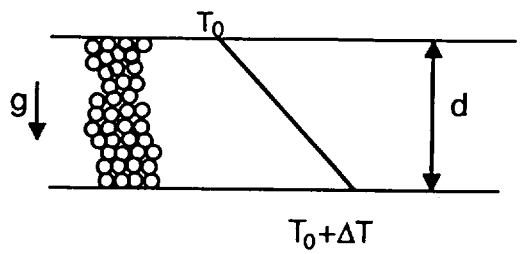

Fig. 1 Schematic diagram of system considered.

where $\mathbf{u}$ is the velocity vector, T the temperature, P the pressure, μ the viscosity, K the permeability, α the effective thermal diffusivity, $\mathbf{g}$ the gravitational acceleration, β the thermal expansion coefficient, ε_s retardation time and λ_s relaxation time. The detailed discussion on physical properties can be found in the work of Katto and Masuoka[3]. The important parameters to describe the present system are the Darcy number Da, the Rayleigh number Ra, and the dimensionless parameters λ

and ε defined by

$$Da=\frac{K}{d^2}\quad Ra=\frac{g\beta\Delta T d^3}{\alpha\nu}\quad \lambda=\frac{\alpha\lambda_s}{d^2}$$

$$\text{and}\quad \varepsilon=\frac{\alpha\varepsilon_s}{d^2} \tag{5}$$

where ν denote the kinematic viscosity. The dimensionless relaxation time λ has the meaning of the Deborah number.

2.2 Perturbation Equations

Under the linear stability theory the disturbances caused by the onset of thermal convection can be formulated, in dimensionless form, in term of the temperature component θ_1 and the vertical velocity component w_1 by decomposing equations (1) ~ (4):

$$\frac{1}{Da}\left(\varepsilon\frac{\partial}{\partial\tau}+1\right)\nabla^2 w_1 = Ra\left(\lambda\frac{\partial}{\partial\tau}+1\right)\nabla_1^2\theta_1 \tag{6}$$

$$\frac{\partial\theta_1}{\partial\tau}-w_1=\nabla^2\theta_1 \tag{7}$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ and $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$. The velocity component has the scale of α/d and the temperature component has the scale of ΔT. The proper boundary conditions are given by

$$w_1=\theta_1=0\quad \text{at}\quad z=0 \text{ and } z=1 \tag{8}$$

The boundary conditions represent no flow through the boundaries and the fixed temperature on the upper and lower boundaries.

III. STABILITY EQUATIONS

3.1 Normal Mode Analysis

According to the normal mode analysis, convective motion is assumed to exhibit the horizontal periodicity (9). Then the perturbed quantities can be expressed as follows:

$$[w_1(\tau,x,y,z),\theta_1(\tau,x,y,z)] = [w_1(z),\theta_1(z)]\exp[i(a_x x+a_y y)+\sigma\tau] \tag{9}$$

where " i " is the imaginary number and σ is the temporal growth rate. With $\mathrm{Re}(\sigma)>0$ the system will become unstable. With $\mathrm{Re}(\sigma)=\mathrm{Im}(\sigma)=0$ the system becomes maginally stable and the critical conditons to mark the onset of stationary motion are well known[1, 2]

$$Ra_{D,c} = 4\pi^2 \tag{10}$$

However, in addition to the above exchange of stabilities, periodic instabilities can be occur with viscoelastic fluid even though $\mathrm{Re}(\sigma)=0$. To analyze this oscillatory instabilty the amplitude functions, w_1 and θ_1 in equations (6) ~ (8) are set as

$$w_1=w_r+iw_i \text{ and } \theta_1=\theta_r+i\theta_i \tag{11}$$

Substituting the above equation (11) into equations (6) and (7) produces the usual amplitude functions in terms of the horizontal wave number $a=(a_x^2+a_y^2)^{1/2}$.

$$(D^2-a^2)w_r = \frac{1}{1+(\varepsilon\sigma_i)^2}\{a^2\sigma_i(\varepsilon-\lambda)\theta_i+a^2(1+\varepsilon\lambda\sigma_i^2)\theta_r\} \quad (12.a)$$

$$(D^2-a^2)w_i = \frac{1}{1+(\varepsilon\sigma_i)^2}\{a^2\sigma_i(\lambda-\varepsilon)\theta_r+a^2(1+\varepsilon\lambda\sigma_i^2)\theta_i\} \quad (12.b)$$

$$(D^2-a^2)\theta_r = Ra_D w_r - \sigma_i\theta_i \quad (13.a)$$

$$(D^2-a^2)\theta_i = Ra_D w_i + \sigma_i\theta_r \quad (13.b)$$

where "D" is the z-directional differential operator, d/dz. The boundary conditions are reduced to

$$w_r = w_i = \theta_r = \theta_i = 0 \quad \text{at} \quad z=0 \quad \text{and} \quad z=1 \quad (14)$$

The objective is to find the value of Ra_L to mark the onset of oscillatory motion for a given a, ε and λ.

3.2 Solution Procedure

To solve the above stability equations the outward shooting scheme is employed. For the integration the upper boundary conditions at $z=1$ must be converted lower ones at z=0 so that the present boundary value problem can be converted to an initial value problem. With the specified values of ε, λ, and a, two eigenvalues Ra_D and σ_i and two initial conditions $D\theta_r(0)$ and $D\theta_i(0)$ are guessed. Since the governing equations and boundary conditions are all homogeneous, the initial conditions $Dw_r(0)$ and $Dw_i(0)$ are assigned arbitrarily.

The integration is performed from $z=0$ to $z=1$ with fourth order Runge-Kutta-Gill method. If the guessed values are all correct, the values of θ_r, θ_i, w_r and w_i should be vanished at z=1. The Newton-Raphson method is used to obtained the better trial values for the next iteration. The computation is repeated untill the desirable accuracy is obtained.

IV. RESULTS AND DISCUSSION

The stability criteria on the porous layer saturated by viscoelastic fluid have been ontained numerically. The stability curves are obtained as function of ε, as shown in Fig. 2. On each curve the minimum Ra_L will be the critical Darcy-Rayleigh number $Ra_{D,c}$ to mark the buoyancy-driven motion. It is known that this porous layer becomes Newtonian with $\lambda=\varepsilon$. For this limiting case the present stability critria coincides with that of Horton and Rogers[1] and Lapwood[2]. The effect of λ on the stability is illustrated in Fig. 3, and it is known that with increasing λ the porous layer becomes more unstable. Detailed stability conditions are summarized in Fig. 4. The present results show that there can exist the overstability of oscillatory motion. The oscillatory motion is favored when $Ra_{D,c} < 4\pi^2$, of which value is given by equation (10).

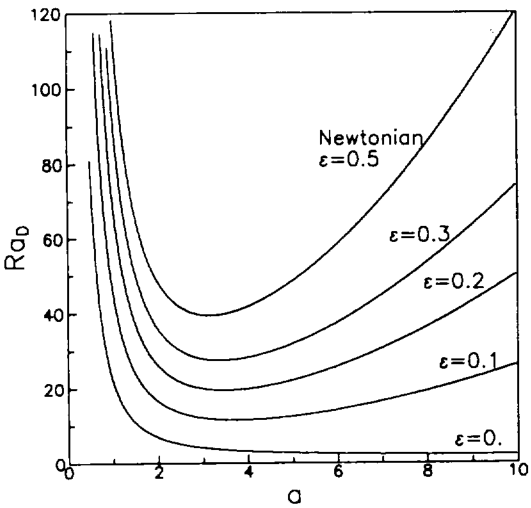

Fig. 2 Neutral stability curves for $\lambda=0.5$

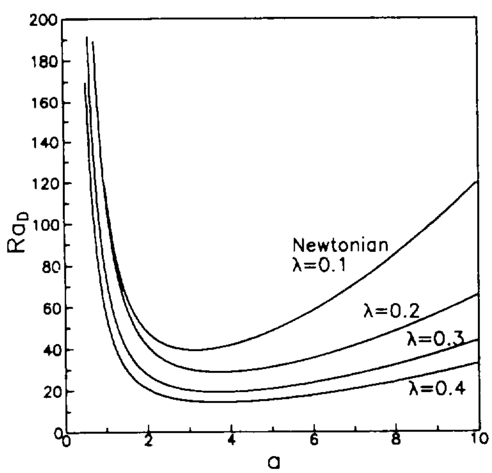

Fig. 3 Neutral stability curves for $\varepsilon=0.1$

Now, it seems evident that viscoelastic fluids exhibit markedly different stability

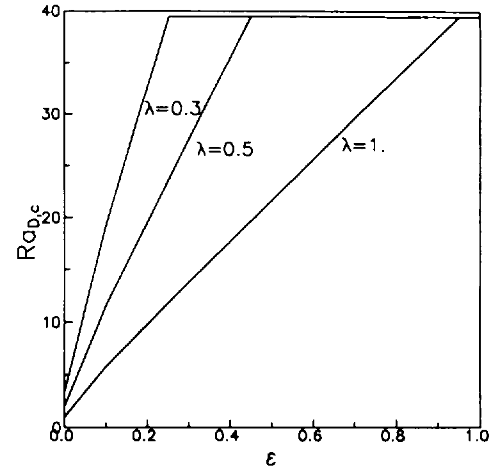

Fig. 4 Critical Rayleigh number vs. ε for various λ

properties as a results of possessing some elasticity. Since ε is related with viscosity and λ cocerns elasticity, the system becomes more unstable with a decrease in ε, but with an increase in λ. It is noted that for the verification of the above statement refined experimental work is needed since no experimental evidence exists as of now.

V. CONCLUSIONS

The onset of buoyancy-driven motion in a horizontal porous layer which is saturated by viscoelastic fluid has been analysed analytically by using linear stability theory. It is known that elasticity parameter is a desstabilizing facror and for a certain parameter range overstability is perferred mode.

VI. 요 약

점탄성 유체로 포화된 초기 정지 상태의 수평

다공질 매질층에서 부력에 의하여 유발되는 열적 불안정성을 이론적으로 해석하였다. 유체의 유동은 수정된 Darcy법칙을 사용하여 해석하였으며, 대류 발생의 임계조건을 찾기위하여 선형 안정성 이론을 사용하였다. 해석결과 점탄성을 나타내는 매개인자의 범위에 따라 overstability가 발생될 수 있음을 보였다.

NOMENCLATURE

a	dimensionless horizontal wave number, $\sqrt{a_x^2+a_y^2}$
d	depth of layer [m]
D	differential operator, d/dz
Da	Darcy number, K/d^2
g	gravitational acceleration [m s $^{-2}$]
K	permeability
P	pressure
Ra	Rayleigh number, $g\beta\Delta T d^3/\alpha\nu$
Ra_D	Darcy-Rayleigh number, $Ra\times Da$
T	temperature
u	velocity vector, [m s $^{-1}$]
u,v,w	dimensionless velocity components of the Cartesian coordinate
x,y,z	dimensionless Cartesian coordinate

Greek symbols

α	thermal diffusivity [m ^{2} s $^{-1}$]
β	volumetric thermal expansion coefficient [K $^{-1}$]
ΔT	temperature difference [K]
ε	retardation parameter
θ	dimensionless temperature
λ	relaxation parameter [s $^{-1}$]
μ	viscosity [Kg/(m s)]
ν	kinematic viscosity [m s $^{-1}$]
ρ	density [kg m $^{-3}$]
σ	temporal growth rate
τ	dimensionless time

Subscripts

c	critical state
i	imaginary part
r	reference state or real part
0	basic state
1	perturbed state

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