Firm–Specific Human Capital and the Behavior of a Firm–A Dynamic Approach

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Ⅰ. Introduction

According to Becker,[1] a distinction must be made between general and firm-specific human capital. Training is, of course, the most important means of investing in human capital, and hence it will be used later as an explicit example of the investmant process. Now if an employee receives general on-the-job training, his productivity will increase wherever he may work after training. On the other hand in the case of firm-specific training, an employee's productivity will increase only in the firm that provides

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1) Becker, Gary S. (1962), *Investment in Human Capital: A Theoretical Analysis*, Journal of Political Economy, 70, no. 5, pt. 2, suppl. : pp.9~49.
———(1975), Human Capital, 2nd ed. (Chicago: The Univ. of Chicago Press)

the training. If he quits or if he is discharged, his productivity falls back to the initial (pretraining) level.

It can easily be shown that a perfectly competitive firm never pays the expenses of general on-the-job training (this is because general training does not bring any return to the firm owing to the fact that in a competitive labor market wages would increase by exactly the same amount as the marginal product). Instead, it is financed by the employee. In this sense, the concept of general training has a meaning identical to that of formal schooling.

As far as firm-specific human capital is concerned, the firm has a more important role to play. In this case the firm pays the cost of training (plus other possible firm-specific investment outlays) and receives the corresponding return in the form of the difference between the employee's marginal producut and the wage rate.

An almost classical treatment of the investment process is found in Becker(1975). In his model investment in human capital occurs during the first period, and the employee in question stays a total of T periods in the firm (there is no labor turnover and human capital depreciation is not explicitly considered). The costs of firm-specific training consist of two parts; namely the opportunity costs of the time spent in training and the direct resource costs. The return from the investment is the difference between the employee's (i.e., trainee's) marginal product and the wage rate. Thus the following equilibrium condition can be written

$$(1)\ MP_o' + \sum_{t=1}^{T-1} \frac{MP_t - W_t}{(1+r)^t} = W_o + C$$

where C is the total cost of training (in the initial period), MP_o' is the opportunity marginal product of trainees, W_o is the wage rate paid during the training period, W_t is the wage rate and MP_t the marginal product in period t, and finally r is the market rate of interest.[2)]

As for the wage rate, W_t, it can be assumed initially that it is the market wage rate. Thus, an employee has no incentive to quit because a higher wage rate could not be obtained elsewhere. On the other hand, the employee loses nothing if he quits. But the

2) All of this follows from the assumption that the firm pays all costs and collects all returns. See Becker (1975), Ibid., p.28.

firm does lose ! The exact amount of the loss depends on the corresponding investment outlay and on the date of quitting. Hence, the employee can use the possibility of quitting as a threat. So in fact what we really need to analyze is a case of a bilateral monopoly.

Thus, in fact, the employee's wage rate will be determined in a bargaining process between the employee and the employer. Obviously the result is that the employee receives a higher wage rate, that is, the market wage rate plus some premium. But the higher wage would make the supply of trainees greater than the demand, and rationing would be required. The final step would be to shift some training costs as well as returns to employees, thereby bringing supply more in line with demand. When the final step is completed the firm no longer pays all training costs nor do they collect all return but they share both with employees. Now if this kind of contractual agreement is reached, the employee suffers a loss if he quits, a fact which obviously discourage quitting.

As for the employer, he suffers a loss if the employee quits or if he discharges the employee. All in all, we see that firm-specific human capital has strong implications regarding labor turnover. It is also clear that firm-specific human capital affects the wage determination process, although it is difficult to discover the exact forms and magnitude of these effects.[3)]

In the following analysis, we consider the accumulation of firm-specific human capital by using a dynamic model of on-the-job training. By employing this kind of analysis it becomes possible to derive the optimal investment path for the firm and to study how investment adjusts in response to changes in different parameters. Our model also provides a suitable framework for analyzing what kind of differences exist between 1) young and old employees, and 2) the employees with different initial human capital endowment (e.g., different level of schooling), especially with respect to investment opportunities and bargaining power. In other words, our model provides what kind of behavior of a firm can be expected in selecting the joint investment (firm-specific investment) partner.

One thing to mention is that this paper concerns only the behavior of a firm in the investmnet in the firm-specific human capital and does not consider .the behavior of an employee. That is, this paper excludes the response of the worker whether to join the

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2) In the competitive equlibrium, the wage rate will be the initial wage (market wage rate) plus the employee's share of the return from the investment. That is, $W_o^+ = W_0 + a(MP' + MP_0)$, where a is the employee's share of the invesment in firm-specific human capital. For more detail, see Becker(1975), and Hashimoto(1979).

firm-specific investment.

Ⅱ. The Optimal Accumulation of Firm-Specific Human Capital

When analyzing the accumulation of human capital, there are basically two approaches to be followed. That presented by Rosen (1972)[4] is based on the hypothesis that learning and work are complementary joint products, the ratio between the varying from job to job. Those work activities involving greater learning content pay lower wages both to cover costs associated with learning incurred by firms and to ration off eager demanders of such jobs. The other approach, presented by Ben-Porath(1967)[5] is characterized by allocation of time between human capital production and work.

Even if the approach of Rosen has some appealing features, it has not gained much popularity, perhaps because of some problems with its implementation. The basic weakness in the model of Rosen is known that, in short, a firm has no means of influencing the accumulation of human capital. That is, Rosen uses the following equation of motion for the stock of firm-specific human capital : $dZ/dt=bQ(t)$, where Z stands for the stock of human capital, Q the index of output, and b a constant parameter. In this model, the firm is really no more than an on-looker, firm-specific human capital being a pure windfall for the firm in question. Moreover, there are no adjustment costs involved in accumulating human capital (Hence a production function with decreasing returns to scale is required to achieve a uniqe steady state). In addition, the rate of labor turnover is no importance whatsoever-clearly not a very realistic assumption. In this sense, the subsequent analysis also follows the other tradition, that is, our model is analogous to the allocation of time model.

Ⅱ.1. The Specification of the Model

These models are usually specified so that an individual is assumed to devote a frac-

4) Rosen, S. (1972), *Learning by Experience as a Joint Production*, Quarterly Journal of Economics, pp. 366~382.

5) Ben-Porath, Y. (1967), *The Production of Human Capital and the Life-Cycle of Earnings*, Journal of Political Economy, 75(4), pp. 353~365.

tion of his time to learning and a fraction to working. Human capital is measured in relation to the learning (or training) time.

Before our model is presented in detail, the main assumptions on which the subsequent analysis is based are given as follows. They are :

Assumption 1(A1) : The stock of labor is constant.

Assumption 2(A2) : There is no uncertainty, that is, the employer knows, for example, all the characteristics of an employee.

Assumption 3(A3) : Wages are given to the employer.

The main elements of our model are the production function of human capital and the production function of marketable goods. As stated above, we assume that the quantity of human capital produced depends on the stock of human capital devoted to this production process. Thus, we can write the following equation of motion for the accumlation of human capital :

(2) $\dot{K}(t)=g(h(t))-vK(t)$

where K(t) stands for the total stock oh human capital, h(t) the stock of human capital devoted to human capital production, v the human capital depreciation parameter and $g(\cdot)$ the production function of human capital. K(t) consists of two parts, h(t) and k(t), where k(t) denotes the stock of human capital devoted to working (i.e., production of marketable goods). The function $g(\cdot)$ is assumed to be concave with $g'>0$ and $g''<0$.

The production function of marketable goods can be written simply in the following form :

(3) $Q(t)=f(k(t))$

where Q denotes the index of output. It is also assumed that $f(\cdot)$ is concave, but not necessarily strictly, with $f'>0$ and $f''\leq 0$.

Since $K=h+k$, we can express $h/K=s$ and $k/K=(1-s)$. Thus we can rewrite (2) and (3) in the form :

(2′) $\dot{K}(t)=g(s(t)K(t))-vK(t)$

(3′) $Q(t)=f((1-s(t))K(t))$.

By substituing (2′) into (3′) we obtain : [6]

(4) $Q(t)=f((1-g^{-1}(\dot{K}(t)+vK(t)/K(t))K(t))$.

From the previous assumptions about f(·) and g(·), we find that $\partial Q/\partial\dot{K}<0$ and $\partial^2 Q/\partial\dot{K}^2<0$,[7] a result which is typical for all adjustment cost models.

As mentioned above in A1, the stock of labor is constant in this model. For simplicity, let us assume that there is only one employee in the firm. In this case, the expected duration of employment is finite, say, T, and the stock of human capital, K, depreciates at a constant rate, v.[8]

The employee has an initial stock of human capital, denoted by K_0 which is completely general in character (if the employee had also acquired firm-specific capital from other firms, this would, by definition, have zero production effect). Instead we assume that all the human capital accumulated in this firm is firm-specific.[9]

By A3, we assume that the firm pays a constant wage rate, say w_o^+. It can be thought to correspond to the market wage rate plus some premium which is a kind of remuneration for the firm-specific capital, as shown in footnote 4), i.e., $W_o^+=W_0+\alpha(MP'-MP_0)$.

Using the previous assumptions, a dynaminc model is now developed.[10] We adopt the conventionl point of departure, namely, that the firm maximizes its present value (here, per employee), that is :

6) from (2′), $K(t)+vK(t)=g[s(t)k(t)]$. Assuming that there exists the inverse function, we can express (2′) as below. $s(t)K(t)=g^{-1}[K(t)+vK(t)]$ ∴ $s(t)=g^{-1}[K(t)+vK(t)]/K(t)$. Thus, by replacing this into (3′), we obtain (4)

7) This can be shown easily by using the differentiability theorem; $(g^{-1})'(x)=1/g'(g^{-1}(x))$. For more detail, See Salas, S.L. & Hille, E. (1974), Calculus, John Wiley & Sons, Inc., A8.

8) If the stock of labor were L (L>1), then, of course, the natural withdrawal of labor would also reduce the stock of firm-specific human capital in the firm; that is, v would now have a different interpretation compared to the one above. In this case, the time horizon of the firm would be infinite.

9) This is a very restrictive assumption. In fact, both kinds of human capital would be produced in a firm.

10) For more detail of the process and the results, consult Intriligator M. (1971), Mathematical Optimization and Eonomoic Theory, Prentice-Hall, N. J. pp.344~357.

(5) $\max \int_0^T e^{-rt}[pf((1-s(t))K(t))-w_o^+]dt$

subject to $\dot{K}=dK/dt=g(s(t)K(t))-vK(t)$

and $0 \le s(t) \le 1$, $K(0)=K_0$, $K(T)$ is free.

The notation has the following interpretation :

−p is the price of output, the firm is a price taker

−w_o^+ is the wage rate

−r is the market rate of interest, here we assume that the firm has static expectations with respect to p, w_o^+ , and r

−v is the human capital depreciation parameter

−f(·) is the production function of marketable goods, Q, it is assumed to be concave with $f'>0$, $f''<0$, $f'(0)=\infty$, and $f'(\infty)=0$

−g(·) is the production function of human capital (per employee) we assume concavity with $g'>0$, $g''<0$, $g'(0)=\infty$, and $g'(\infty)=0$

−K(t) is the stock of human capital

−s(t) is the fraction of human capital used in human capital production, thus 1−s corresponds, in fact, to the intensity of working

−T is the duration of employment

(5) is a typical control problem with K the state variable and s the control variable. Next we develop a current value Hamiltonian and then write out the necessary and sufficient conditions for the optimal path of K(t). The Hamiltoninan corresponding to (5) is :

(6) $H(K,s,y,t)=e^{-rt}[pf((1-s)K)-w_o^+]+y(g(sK)-vK)$.

Now we just replace the costate variable y[11] by ue^{-rt} so that we get

(6′) $H(K,s,u,t)=e^{-rt}[pf((1-s)K-w_o^+]+ue^{-rt}(g(sK)-vK)$.

And the current value Hamiltonian $\bar{H}=H\,e^{rt}$, i. e.

(7) $\bar{H}(K,s,u,t)=pf((1-s)K-w_o^+)+u(g(sK)-vK)$.

11) This is the dynamic equivalent of the Lagrange multiplier of static problems of maximization (or minimization) subject to constraints.

The necessary conditions corresponding to (7) are now : [12]

(8) $\dot{u}=ru+uv-pf'(1-s)-ug's=0$

(9) $\dot{K}=g(K)-vK=0$

(10) $\partial H/\partial s=-pf'K+ug'K=0,\ \partial^2H/\partial s^2=pf''K^2+ug''K^2<0$

(11) $u(T)e^{-rt}=0$

The costate variable, u, can be interpreted as the shadow price of human capital,[13] and from (8) and (10) we can solve :

(12) $u(t)=\int_t^T [pf'\ \exp.-(r+v)(\tau-t)]d\tau=\int [pg'\ \exp.(r+v)(\tau-t)]d\tau$

i.e., u(t) is the present value of adding a unit of human capital to the stock of human capital at time (date) t.

The necessary conditions give us the optimal pair, denoted by K(t), s(t). By showing that the current value Hamiltonian H(K, s, u, t) is concave in K and s, we obtain the sufficient condition for K(t), s(t) to be the solution of (5). That is, in order to satisfy the sufficient condition, the following Hessian matrix should show 'negative definite'.

(13) $$\begin{bmatrix} -f''(1-s)^2+ug''s^2 & -f''(1-s)K+ug''sK \\ -f''(1-s)K+ug''sK & f''K^2+ug''K^2 \end{bmatrix}$$

It emerges that the determinant of the matrix is simply $f''g''K^2u$, hence by our previous assumptions the matrix is negative definite.

II.2. The Optimal Accumulation of Firm-Specific Capital

Next we derive the optimal value for the control variable, s. (10) gives us an analogue for the marginal cost = marginal revenue condition, pf'K representing here the opportunity cost term, while ug'K corresponds to the revenue from allocating human

12) If we differentiate $u=ye^{rt}$ with respect to t, we get : $\dot{u}=rye^{rt}+\dot{y}e^{rt}=ru+\dot{y}e^{rt}$. On the other hand, $\dot{y}=-\partial H/\partial K=e^{-rt}(pf'(1-s)+yg's-yv)$. (See Intriligator, Ibid., p.353). Hence, $\dot{u}=ru-e^{rt}(e^{-rt}pf'(1-s)+yg's-yv)$. This, in turn, can be written in the form (8) above.

13) This has the same meaning of the Lagrange multiplier. For example, the multiplier in the utility maximization problem is interpreted as the marginal utility of money.

capital to human capital production. As for the optimal value of s, we only examine the case of an interior solution $0<s<1$ (i.e., $pf' K = ug' K$). Again by using (10), we can obtain the following signs of the partialal derivatives with respect to K and u for s :

(14) $\bar{s} = \bar{s}(K, u)$[14)]

$\qquad -\ ?\ +$

The problem is that the sign of $\bar{s}_k$ is ambiguous, i.e., the possibility cannot be excluded that the fraction of human capital devoted to human capital production increases when the stock of human capital increases. This result would, of course, be a perverse one (if the standard theory of investment is used as a frame of reference). If the stock of human capital increases, however, the marginal productivity of the additional stock of human capital will decrease. Thus, it may be safely said that the increase of K would reduce the fraction of the capital stock devoted to the production of human capital, s (i.e., $\bar{s}_k<0$).

On the other hand, if the production function $f(\cdot)$ is linear, the sign of $\bar{s}_k$ is unambiguously negative.[15)]

As for the sign of $\bar{s}_u$, it is unambiguously positive, which is also intuitively obvious if we recall (12). Because u, in fact, represent (implicit) future profits from the investment in human capital, (10) indicates that the more profitable it is to invest the greater the investment will be.

In order to discover how s(t) and K(t) actually behave over time, we develop a phase diagram. First we write out the equations of motion which correspond to the optimal values of s(t). By substituting (10) and (14) into (8) and (9) we obtain :

(15) $\dot{u} = (r+v)u - pf'\ ((1-\bar{s}(K,u))K) = 0$

(16) $\dot{K} = g(\bar{s}(K,u)K) - vK = 0$

14) $\bar{s}_k$, $\bar{s}_u$ are obtained by the implicit differentiation of (10). They are;

(14-1) $\partial \bar{s}/\partial K = [pf''(1-s) - ug''s]/(pf''K + ug''K) = ?$

$\qquad (-)\quad (+) \qquad\qquad (-)$

(14-2) $\partial \bar{s}/\partial u = -g'/(pg''K + ug''K) > 0$

15) since $f''=0$ when the production function $f(\cdot)$ is linear, (14-1) becomes negative.

At this stage we are only interested in the singularities of (8) and (9). Now the corresponding slopes can be derived by implicit differentiation :

$$(17)\quad du/dK_{|\dot{u}=0}=\frac{pf''(1-\bar{s})-pf''\bar{s}_kK}{r+v+pf''\bar{s}_uK}$$

$$(18)\quad du/dK_{|\dot{K}=0}=\frac{v-g'\bar{s}-g'\bar{s}_kK}{g'\bar{s}_uK}$$

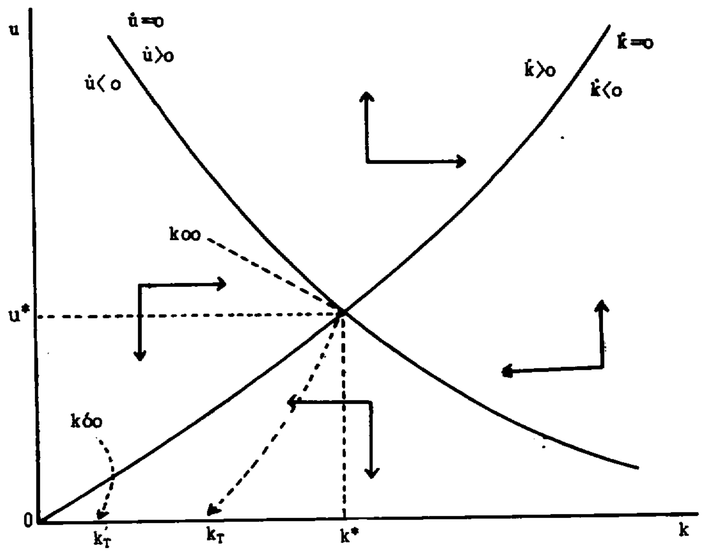
As far as (17) is concerned, at first sight there seems to be ambiguity about the sign. However, if we substitute $\bar{s}_k$ and $\bar{s}_u$ into the numerator and the denominator and solve from the singularity of (8), $r+v=g's+pf'(1-s)/u$, and then use this result to evaluate the sign of the denominator, we find that (17) is unambiguously negative.

In the case of the $\dot{K}=0$ locus, the denominator is clearly positive, but there is some ambiguity about the sign of the numerator. If $\bar{s}_k$ is negative, it is positive as it the whole slope of (18).

The possible explanation of these results are as follows. This can be seen by examining what happens to these equations of motion if either u or K goes to zero or increase by a very large amount. We begin with the $\dot{u}=0$ locus and assume that u approaches 0. Then we can see from (8) that vu+ru approaches 0, too, which implies that $pf'((1-\bar{s})K)$ must go to zero. Because $\bar{s}_u$ is positive, s approaches 0 and $pf'((1-\bar{s})K)$, in turn, $pf'(K)$; for this reason, K must go to infinity in order to keep $\dot{u}=0$. The opposite holds in the sense that if K goes to zero, $pf'((1-\bar{s})K)$ goes to infinity, so that u is required infinity, too. otherwise $\dot{u}<0$. Clearly the locus must have a negative slope.

Going on to the $\dot{K}=0$ locus, we start with the case in which K goes to infinity. Then g(sK) must do the same otherwise K becomes nonzero. Even if g(·) increase with K (because $g'>0$), vK increases faster, since g(·) is assumed to be concave and because $\bar{s}_k$ is presumably negative, moving in the opposite direction from K. If, in turn, K approaches zero, u must do the same, i.e., the locus continues through to the origin. Strictly speaking, K cannot, however, go to infinity, because s cannot exceed 1; hence $g(K)-vK=0$ forms an upper bound for K. In the same way, we find an upper bound for the shadow price of human capital, i.e., u. Because s is restricted by the lower bound s=0, u cannot exceed $pf'(K)/(r+v)$.

Now we are in a position to draw the phase diagram for the equations of motion. [Figure 1] describes this case. The optimal path is illustrated by the curve $K_{\infty}K_T$. It



⟨Figure 1⟩

follows first the infinite time trajectory $K_{\infty}K'_{\infty}$, then falls and reaches the $\dot{u}=0$ co-ordinate (so that the transversality condition (11) is satisfied).16) These optimal paths are based on the assumption that the time horizon, i.e., the duration of employment, is relatively long. However, if the time horizon, T, were short, the optimal path would not approach the steady state, but rather fall rapidly as illustrated by the curves $k'_{\infty}K'_{T}$.

Now we briefly analyze the stability of the system. First we show that K^*, u^* is a saddle point. With this aim in mind, we make the familiar linear expansion of (15) and (16) around the steady state. This operation gives us the following system :

$$(19)\quad \begin{bmatrix} \dot{u} \\ \dot{K} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} u-u^* \\ K-K^* \end{bmatrix}$$

16) If we were analyzing an infinite time problem, the firm would stay in the steady state having once reached it. Then the firm would invest just so much that the investment would equal the loss which is caused by human capital depreciation and the natural withdrawal of labor.

where the a_{ij}'s stand for

$a_{11}=r+v+pf''\bar{s}_u K>0$

$a_{12}=-pf''(1-\bar{s})+pf''\bar{s}_k K>0$

$a_{21}=g'\bar{s}_u K>0$

$a_{22}=v+g'\bar{s}+g'\bar{s}_k K<0$

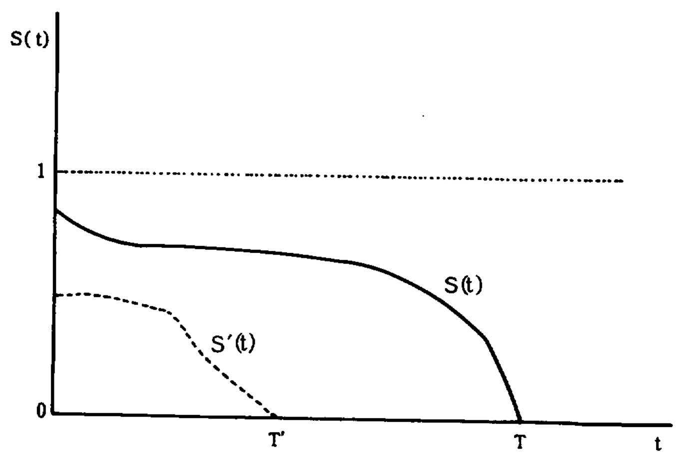
The characteristics roots of the matrix A are:

$$(20)\ \lambda_{1|2}=\frac{-(a_{11}+a_{22})\pm\sqrt{(a_{11}+a_{22})^2-4(a_{11}a_{22}-a_{12}a_{21})}}{2}$$

Because the term $(a_{11}a_{22}-a_{12}a_{21})$ is negative, the roots are real but opposite in sign. Therefore K^*, u^* is a saddle point. That is, there is only one path in the (K, u) space converging on the equilibrium (i.e., $K_{\infty}K_T$).

II.3. The Time Path of s(t) and K(t) – Firm's Optimal Policy

By referring to the previous diagram (figure 1), we can now derive the time paths of s(t) and K(t) which correspond to the firm's optimal policy. (Figure 2) illustrates the time paths of s(t) in the case of a finite time horizon. s(t) corresponds to the trajectory $K_{\infty}K_T$ and s' to $K'_{\infty}K'_T$. If f(·) is strictly concave and we can exclude the



(Figure 2)

possibility that the price of output,. p, is temporarily zero, s(t) would not meet the upper bound, s=1.

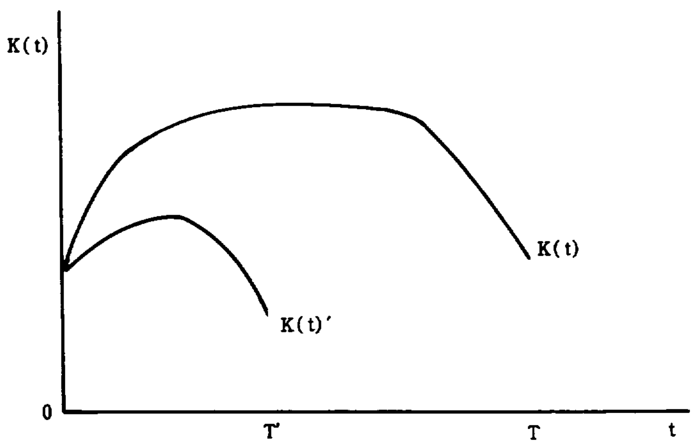
The time path of K(t) can now be derived from (16) so that we get :

$$(21)\ K(t)=\int_0^t [g(\bar{s}(\tau))\ \exp.(v(\tau-t))]d\tau-K_0\ \exp.^{-vt}$$

(Figure 3) llustrates the time path of K(t) corresponding to the time path of s(t) in (Figure 2) ; K′(t) is similary related to s′(t). If the time horizon is "long enough", the time path of K(t) has a typical turnpike property, i.e., K(t) is kept near the steady stat, K*, most of time.

Some obvious conclusions can be made on the basis of these figures. First of all, we see that those employees who will have only a short employment horizon, T, in the firm (at the moment they are hired), receive only a limited amount of training. In other words, (12) implies that the shorter is T, the smaller is u, and, because $\bar{s}_u$ is positive, the smaller is s(t).

In this sense, old workers, in particular, face gloomy prospects in the labor market; they will not find a firm which will provide them with a training program of any note, and the less training they receive, the less prospect they are against layoffs, for example.[17] This reasoning is equally applicable to the fact that a firm prefers a man to



(Figure 3)

17) The old workers have not only a shorter T but presumably also a smaller g(·), i.e., the ability to produce human capital; the consequence of this becomes apparent in Table 1 below. By definition, the fast quitters suffer the same fate as the old workers; The only problem is that, in practice, a firm cannot accurately identify these employees.

a woman in providing the investment opportunity. Especially this is more apparent in Korea where most of female workers leave the labor force after marriage and, thus, they have a very short employment horizon (T).

On the other hand, if we consider the situation inside a firm, we notice, on the basis of (21), that the typical situation is (ceteris paribus) that the longer the employees have stayed in the firm, the larger is their stock of human capital.

Clearly the incumbent employees (i.e., the employees who have been in the firm for some time) are in a better position than the new entrants who have just been hired, because the former possess a greater stock of firm-specific human capital.[18] Thus, if they are fired or if they quit, the firm suffers a greater loss than with new employees. On the other hand, in the labor market there are various seniority rules, which can be seen in the layoff practice and in the wage differentials. It seems clear that the pattern of human capital accumulation could provide an explanation why firms more or less follow these rules.[19]

Next we consider the effect of differences in levels of K_0, i.e., in the initial endowments of human capital. For example, it can be asked whether it is possible that in some period t employees with different initial endowments of human capital, K_0, have the same $K(t)$ (assuming that they are identical in other respects, i.e., have the same production fuctions, time horizons, etc). According to the uniquness theorem of differential equations (cf. Takayama (1974), p.305) this is impossible, since if the paths, say, for example, $K_{01}(t)$ and $K_{02}(t)$ cross, they must be identical through the interval [0, T]. Hence an employee with a lower initial endowment of human capital will never catch up with another employee with more human capital in period $t=0$. That is, if an employee has an initial advantage over another (for example, in the sense that he has more formal schooling) he will never relinquish this advantage.

18) In this connection, we refer to Figure 3, especially to the time path $K(t)$. It should be noted that the employees near the retirement age (t near T) have a relatively small stock of human capital, so that even if K typically increases with the time an employee has been in the firm, there is an exception concerning "very old" employees.

19) As far as wage differentials are concerned, they will be discussed later in another paper. As for layoffs, we can refer here to the results obtained by Becker(1975) and Oi(1962), namely, that is, there is an inverse relationship between the stock of human capital and the layoff possibility. This inverse relationship has been found in many empirical analyses. See, for example, Oi(1962) and Telser(1973).

Ⅲ. Effect of Changes in the Demand and Wage Rate

Now we take a look at what happens to the steady state stock of human capital (K^*) and the present value of adding a unit of human capital to the stock of human capital (u^*) (around the steady state), when there are changes in the wage rate and demand. The comparative statics results concerns the effect for cetris paribus once-and-forever change in the respective parameter. Here we examine, however, the effects of the permanent shift and temporary change in the parameters in question. The results are summarized in Table 1.

Ⅲ.1. Change In The Wage Rate

The change in the wage rate (w_0^+) has, in short, no effect on u^* and K^* regardless whether the change is permanent or temporary if other things are equal. The reason is that w_0^+ simply represents a given lump-sum cost for the firm which does not affect the production process. That is, under our assumptions A1 (constant stock of labor) and A3 (given wage rate), the increase in w_0^+ only raises the lump-sum cost for the firm.[20]

〈Table. 1〉

Parameter Increased	dU^*	dK^*
Permanent Change		
W_0^+	0	0
P	+	0
Temporary Change		
W_0^+	0	0
P	0	-

Ⅲ.2. Change In The Demand

Unlike the case of the wage rate change, the effects of change in the demand for the

20) For more detsail, see equations (6), (12) and (21).

output are not same when the change is permanent and temporary. The reasons are as follows.

First, the permanent increase of the demand (i.e., the rise of p) raises the shadow price of human capital (increase in u^*), but need not increase the steady state stock of human capital since it also increases the opportunity costs. This "neutrality" result is already apparent from (10) and (12), i.e., p does not affect s.

Second, if the shift were of a temporary nature, so that after a short period the price returns to the level which corresponds, for example, to a "normal" long-run state, the following results could be obtained. The temporary increase in the firm's (output) demand would make the shadow price of human capital rather insensitive (see (12) on the presumption that T is large enough). But since an increase in p implies higher (opportunity) costs in accumulating human capital, less fraction of human capital would be deveoted to the production of human capital and this, in turn, decrease the stock of human capital. On the other hand, the production of the marketable good would be increased.

Some interesting result is obtained when there is a temporary decrease in the price of output. The shadwo price of human capital will be insensitive and the opportunity costs in the human capital production lower in this case by the reasons given above. Because (10) or (14) implies that s is negatively related to K but positively to u, s will increase and the capital stock, too. This result is obvious if we notice that the firm has, in fact, two production processes; production of marketable goods and human capital. A temporary decrease in p makes human capital production more profitable and thus the firm allocates its production accordingly. This adjustment possibility makes it easier for the firm to withstand the variability of output prices (i.e., demand).

Perhaps the main implication of firm-specific human capital is that it prevents layoffs (cf, for example, Oi (1962), p.554). The previous analysis suggests, however, that the firm has some adjustment possibilities via the allocation of human capital to different purposes so that the (opportunity) costs of not laying off employees can be reduced.

Ⅲ.3. The Role of Wage

The previous analysis is based on the assumption that the wage rate, w_0^* , is simply a given constant for the firm. For this reason, the wage rate does not play any role in the previous analysis. This is hardly a very realistic state of affairs. However, w_0^* would, of course, begin to assert some influence if we introduced a periodic constraint for profits, that is, $pf[(1-s(t)K(t)]-w_0^* \geq 0$. In this case, w_0^* would affect the investment process; for example, the constraint would rule out the corner solution $s=1$, i.e., the firm would never specialize in the production of human capital. Because the unconstrained case is characterized by high value of $s(t)$ and low value of $K(t)$ at the beginning of the training period (assuming that a typical case, $K_0<K^*$, is being considered), the constraint would be effective during the first "periods", smoothing the time pat of $s(t)$.

Rather than being a given constant, w_0^* represents an instrument for the firm. At this stage we cannot, however, treat w_0^* as the firm's (optimal) control variable in a straightforward manner since we do not know all the elements of the corresponding optimization process. This particulary is true with the labor supply constraint (especially the quit function); and if such a constraint is not introduced into the model, the firm would not pay a higher wage than the market wage, w_0. It has already been suggested in section I that, in fact, wages are determined in a bargaining process between the employee and the employer. The employee's asset is then his threat to quit and thus to cause a loss to the employer (and, in fact, to the employee himself, if there are any wage premiums and transfer costs). Thus the bargaining power of the employee depends on the value of the firm-specific human capital he destroys by leaving the firm. By definition, the value, in turn, depends on the stock of firm-specific human capital and its (shadow) price. The latter is, cf., (12), primarily determined by the length of time an employee stays in the firm, i.e., T-t.

Now it seems justified to study a wage function, $w_0^*=w_0^*(K(t),t)$, as a firsthand approximation for the result of the bargaining process (the other result is that the employee does not quit). In the following analysis we assume that $w_0^*(K(t),t)$ is separable and may thus be written $w_0^*=w(K(t))b(t)$. First, we study the case $b(t)=1$.

If we use our model (5) as a frame of reference, it can be seen that wages do not directly affect the determination of the control variable, $s(t)$, whereas they have an effect on the costate variable $u(t)$. In particular, the equation of motion (15) must now be written in the form :

(22) $\dot{u}=(r+v)u-pf'((1-s))K)+w'=0$

Now wages affect both the steady state and the adjustment to the steady state (note the the slope of the optimal trajectory is simply $du/dK=\dot{u}/\dot{K}$). We consider first the adjustment process. In the phase diagram (Figure 1), the slope of the $\dot{u}=0$ locus was nonpositive. In order to obtain the same result also in the case where w_0^*, the second derivative of $w(\cdot)$ must be nonnegative. Namely, if $w''<0$, i.e., the growth of wages decreases with K, there is ambiguity as to the sign of the slope of $\dot{u}=0$, that is, the possibility of a positive slope cannot be ruled out. this, in trun, makes it possible that s(t) increases over time. Presumably this perverse result does not arise, and it definitely does not arise, if $w''>0$ as stated above. Thus, [Figure 2], which illustrates the time path of s(t), is also relevant in this case where $w_0^*=w(K)$.

With regard to the steady state, the following comparative statics results for cetris paribus once-and-forever increase in wage and price parameters are considered.

〈Table. 2〉

Parameter Increased	du^*	dK^*
P	+	+
w′	-	-
w (w′ constant)	0	0

Table 2 shows that the level of wages does not matter even in this case; only the slope of the wage function affects the stock of human capital and the corresponding shadow price. According to Table 2, an increase in the price of output, p, increases the steady state stock of human capital, K^*, whereas according to Table 1, p has no effect on K^*. there is an obvious reason for this difference : if the wage depends on the stock of firm specific human capital, an increase in p, while increasing the opportunity cost of producing human capital and increasing the return from human capital also decreases the "real" cost of acquiring firm-specific human capital due to the fact that w'/p, the "real" (marginal) wage rate, decreases.

An increase in the price of output, p, increases the firm's output in the long run. this is simply the consequence of the increase in K^*.

In the short run the situation is not so simple. For, if there is a once-and-forever increase in p, this immediately increases the shadow price of human capital, u, which in turn, implies a higher value for s. Because the production function is of the form $Q=f((1-s)K)$ and because the stock of human capital can be increased only over time, we obtain a perverse production effect : If p increases (permanently), the immediate effect is a decrease in Q. That is, given the model, the firm has a downward sloping supply curve in the short run.

This result is intuitively obvious, for in this model human capital is, in fact, the only factor of production, and, moreover, a firm can increase its stock of human capital only by taking resourses away from the production of marketable goods.

Finally, we briefly discuss the case in which there is a wage function of the form $w_n^* = w(K(t))b(t)$. With reference to the previous discussion, the assumptin is now made that $b(t) \leq 0$, i.e., cetribus paribus, an employee's bargaining power decreases over time.

The fact that b decreases over time affects the shadow price of human capital, u, and thus also the investment in that capital. If we rewrite equation (22),

$$(22') \quad \dot{u} = (r+v)u - pf'((1-\bar{s})K) + w'(K)b = 0$$

we find that if b decreases over time, this implies a higher value for u and thus a higher investment rate, s(t). Now (12) can be written as :

$$(12') \quad u(t) = \int [pf'((1-\bar{s}(\tau))K(\tau)) - w'(K(\tau))b(\tau) \ \exp.(r+v)(t-\tau]d\tau$$

Thus, when t increases (decreasing the remaining duration of employment, T-t), the fall in b(t) slows down the deterioration in the shadow price of human capital, u. Hence, investment in firm-specific human capital becomes less disadvantageous towards the end of the period of employment, T. this fact will, of course, affect the time paths of s(t) and K(t). Referring to Figures 2 and 3, we can state that the main result is that the peak in the stock of human capital is attained at a later point, in time, i.e., at higher values of t.

Ⅳ. Conclusions and Further Studies Suggested

In this study we analyze a firm's optimal accumulation of firm-specific human capital and labor. Dynamic models are used to extend the results of Becker and Oi. Section Ⅰ introduces the basic concepts and issues, in Section Ⅱ we derive the optimal investment path in relation to firm-specific human capital in the case of a constant stock of employment.

The following main results are obtained.

First, it is shown that investment in firm-specific human capital can be viewed analogously to investment in physical capital, in general. That is, firm-specific capital makes labor more like physical capital, i.e., labor becomes a "quasi-fixed" factor of production.

Second, the time path of firm-specific human capital obtained in this analysis also shows a basis for explaining the seniority rules among employees. A similar implication derived in this analysis is that a firm prefers younger workers to older workers in selecting this joint investment project, if other things are equal. It also shows that this can be equally applicable to the reason why a firm is reluctant to invest jointly with female workers. Another interesting result obtained through this analysis is that, if an employee has an initial advantage of the stock of human caital over another (for example, more formal schooling), he will never relinquish this advantage. This may explain why people want to get more education despite the increasing cost of education.

Third, as for the investment path, it is shown that the wage rate and the price of output affect this time path in a way which is crucially dependent on the relationship between the wage rate and the employee's stock of firm-specific human capital. It is also shown that if there are temporary changes in the firm' s (output) demand, the firm adjusts the investment process, thus being able to compensate partly for the loss of not laying off the employee(s).

Fourth, we show that in the short run there can be perverse production effects with regard to the unexpected changes in price of output : for example, the possibility of a downward sloping supply curve cannot be excluded, even though this result is derived under a very restrictive assumption that labor is the only factor of production.

In short, it emerges that labor with firm-specific human capital differs from "raw" labor, and the difference is especially striking if a temporary change in output demand is considered.

Despite all the merits derived from this simple dynamic model, this study also has many shortcomings. They are as follows.

First, this study considers a single firm only. Hence, this model can produce only first-hand results which do not take into account the "industry effect" which results from the fact that the chages of different variables are correlated over firms. Thus, this analysis should be considered as a first step towards a more complete general equlibrium model.

Second, This study considers only the behavior of a firm in the investment in firm-specific human capital. It is already mentioned that, however, this investment is made jointly between the employer and the employee(s). thus, we should consider the behavior of the employee whether to participate in the joint investment at the same time. By so doing, we will be able to draw a better picture on the nature and importance of firm-specific human capital.

References

Becker, Gary. S. (1962)), *Investment in Human Capital: A Theoretical Analysis*, Journal of Political Economy 70, no. 5, pt. 2, suppl. : pp. 9~49.

———(1975), Human Capital, 2nd ed. The Univ. of Chicago Press, Chicago.

Ben-Porath, Y. (1967), *The Production of Human Capital and the Life-Cycle of Earnings*, Journal of Political Economy 75 : pp. 352~365.

Oi, Walter (1962), *Labor as a Quasi-Fixed Factor of production*, Journal of Political Economy 70(6) : pp. 538~55.

Hashimoto, M. (1979), *Bonus Payment, on-the-job Training, and Lifetime Employment in Japan*, Journal of Political Economy 87(5) : pp. 1086~1104.

Rosen, S. (1972), *Learning by Experience as a Joint Production*, Quarterly Journal of Economics : pp. 366~382.

Intriligator, M. (1984), Mathematical Optimization and Economic theory, Prentice-Hall, N. J.

Salas, S. L. & Hille, E. (1974), Calculus, John Wiley & Sons, Inc.

Takayam, A. (1974), Mathematical Economics, Dryden Press.

Telser, L. (1973), Competition, Collusion and Game Theory, Aldine.

〈국문초록〉

기업특수적 인적자본과 기업의 행태-동태적 접근방법

고 필 수

인적자본(human capital)이란 인간에 대한 교육, 직장훈련등의 투자에 의하여 그 경제가치 또는 생산력의 크기를 증가시킬 수 있는 자본으로 정의된다. 이러한 의미에서 투자에 소요되는 지출이 있으면 그에 상응하는 수익이 발생하는 실물자본(physical capital)의 개념과 다를 바가 없다. 인적자본은 크게 두가지로 분류되는데, 하나는 일반적 인적자본(general human capital)이고 다른 하나는 기업특수적 인적자본(firm-specific human capital)이다. 여기서 일반적 인적자본이라 하면 어떠한 기업에서도 그 유용성이 똑같은 자본을 가리키는데, 그 유용성의 일반성 때문에 그 비용 또한 수익자인 근로자가 모두 부담한다. 반면 기업특수적 인적자본은 그 자본 또는 기술을 습득한 기업에서만 유용하고, 다른 기업에서는 전혀 쓸모없는 자본을 말한다. 이 경우 해고나 자진사퇴시 그 자본은 상실되는데, 그 상실가능성을 감소시키기 위하여 궁극적으로는 기업측과 근로자가 투자비용의 공동부담과 투자수익도 비용부담율에 따라 배분하는 합작투자를 하게 된다. 그때문에 근로자와 기업은 각각 실물자본에 대한 투자와 마찬가지로 기업특수적 인적자본의 최적축적을 모색하게 된다. 이 논문에서는 오직 기업의 형태에 관해서만 동태적인 분석방법을 이용하여 살펴보았다.

그 연구 결과는 아래와 같이 요약된다.

첫째, 일반적으로 기업특수적 인적자본은 노동을 실물자본과 같은 성격을 가지게 한다. 즉 노동은 한 기업에 있어서 준고정요소(quasi-fixed factor)로서의 역할을 한다.

둘째, 기업특수적 인적자본축적의 시간경로(time path)는 기업이 왜 연공서열(seniority rule)을 중요시 하는가를 설명해주고 있다. 이와 관련된 결과로서 기업은 왜 합작투자의 대상자로 나이가 많은 근로자보다 나이가 적은 근로자, 여자보다는 남자근로자를 선호하는가를 도출할 수 있었다. 이 연구를 통해 얻을 수 있는 한 가지 흥미있는 것은, 입사시 한 근로자가 다른 근로자보다 많은 양의 일반적 인적자본을 보유하고 있다면, 전자는 기업특수적 인적자본축적에서도 그 이점을 빼앗기지 않는다는 점이다. 이는 곧 자본에 대한 수익, 즉 임금수준은 전자가 후자보다 항상 높다는 것을 의미한다. 이 결과는 고학력으로 갈수록 교육비의 상승폭이 큼에도 불구하고 사람들이 왜 보다 많은 교육을 받으려고 하는가를 잘 설명해 주고 있다.

셋째, 여러 비교정태분석중 가장 특이한 점은 기업제품에 대한 수요가 일시적으로 감소했을 때

얻은 결론이다. 이러한 경우 기업은 인적자본에 대한 투자비중을 증가시키고, 제품생산에 투입되는 자본량을 감소시킴으로써 수요의 감소로 인한 매출량(생산량)의 감소에 적응할 수 있다. 그 때문에 기업은 기업특수적 인적자본을 보유하고 있는 근로자를 해고시키지 않고 그 자본의 상실을 극소화할 수 있고, 그 근로자는 고용의 안정성을 이룰 수 있다. 이것은 수요가 감소하면 즉각적으로 근로자를 해고시킨다는 기초적인 이론과는 어느 정도 상충하는 것이다. 그러나 실제의 상황과 비교할 때, 이 모형이 보다 많은 설득력을 가진다고 볼 수 있다.

가장 단순한 형태의 동태적 모형을 이용하여서도 위와 같이 많은 현상을 설명할 수 있었지만, 이 모형 역시 많은 단점들이 있다. 그렇기 때문에 그 단점들을 보완하기 의한 연구가 더 진행되어야 할 것이다. 그 들중 두가지만 지적하면 다음과 같다.

첫째, 이 연구의 대상이 한 기업에 국한되었다는 점이다. 즉 여러 변수들이 변할 때 산업전체에도 영향이 미치고, 그것이 그 산업내의 기업에도 영향이 있겠지만 이를 무시하였다. 이러한 이유에서 이 모형은 보다 완전한 일반균형모형을 위한 첫 단계로 간주되어야 할 것이다.

둘째, 위에서도 지적한 바와 같이, 이 연구는 기업의 형태에 대해서만 국한시켰다. 그러나 기업특수적 인적자본투자가 기업과 근로자의 합작으로 이루어지기 때문에, 다른 투자의 주체인 근로자의 행태분석도 있어야 할 것이다. 그렇게 함으로써, 기업특수적 인적자본의 역할과 그 중요성에 대한 보다 좋은 그림을 그릴 수 있을 것이다.