Repeated Games with Imperfect Private Monitoring: A Review Article

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CONTENNTS

I. Introduction

In this review article, we have presented an illustrative introduction to repeated games with imperfect private monitoring. Overall reveiw about recent contributions on private monitoring in repeated games was done by Kandori(2002). In this paper we review the papers in this area in lecture based style. In one shot static games such as prisoners' dilemma game, strategic competition may result in a bad outcome which is pareto inefficient. But the repeated play of stage game may illicit cooperative outcome which is pareto efficient. Thus the theory of repeated games provides a formal framework to explore the possible cooperation in ongoing interactions, such as collusion between firms, cooperation among workers, and international policy coordination.

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The extensive literature has now established traditional Folk Theorems of efficiency. However, virtually all those existing results rely on the crucial assumption that players share common information about each other's actions. This review article provides an illustrative overview of recent growing literature which relaxes this restrictive assumption. In the perfect monitoring case (for example, the Folk Theorem by Fudenberg and Maskin (1986)), the action profiles observed by players are common knowledge.

Players observe a common public signal in each period which is a imperfect indicator of action profiles taken in the current period in the imperfect monitoring case explored by the majority of the existing literature (for example, Green and Porter (1984), Abreu, Pearce and Stacchetti (1990) and Fudenberg, Levine and Maskin (1994)).

These perfect or imperfect public monitoring cases are very restrictive in practical applications to usual economic problems. For example, consider the situation of firms' offering secret price cutting to their customers. The firms cannot observe others' secret offers, but can conjecture those offers based on the their sales. The imperfect private signals are the pairs of individual firm's secret price offer and its secret sales: the outcome in each period is not commonly observed. That is, there are no common signals shared among players. In this case we have a repeated game with imperfect private monitoring. As we know, in the Green and Porter's model, the common public signal is a price which is a stochastic function of unobserved outputs chosen by firms.

The literature about repeated games with imperfect public monitoring characterized the Perfect public equilibria which had a very nice recursive structure. But what happens if the outcome in each period is not commonly observed? In this case we lose a recursive structure to easily characterize equilibria in repeated games. So, relatively little is known about the structure of equilibria in these games. The objective of this paper surveys a few recent idea on the topic.

In this review paper we first consider a simple example of private monitoring. In two period of repeated play we can enforce cooperation in the first period if private monitoring is highly, but not perfectly correlated. Even if private monitoring is independent between players, Cooperation can be enforced in the first period if players use mixed strategies. Secondly we reviewed Sekiguchi (1997)'s remarkable work of private monitoring in the infinitely repeated play. We also reviewed Piccione (2002), Ely and Valimaki (2002)' ongoing randomization to build up the Folk Theorem under almost perfect private monitoring. We conclude this paper by commenting on the related several works in this area.

Ⅱ. Private Monitoring : a Simple Example

Following Mailath and Morris (1998) and Bhaskar and Van Damme (2002), let's work through some variations on a simple two-period model to get a sense of the problems arising when there is imperfect private monitoring. There are two players who play a prisoner's dilemma game in the first period :

Player Ⅱ / Player Ⅰ	C	D
C	1,1	-1,2
D	2,-1	0,0

In the second period, they play a coordination game

Player Ⅱ / Player Ⅰ	G	B
G	k,k	0,0
B	0,0	1,1

We assume that $k \geq 2$ and there is no discounting : all the payoffs accrue after the second period. This game looks like the repeated game in the sense that the players can try to enforce cooperation in the first period by coordinating on either a good or bad outcome on later.

2.1 Perfect Monitoring Case

In the above two period game, we can show that the first period cooperation will be supported by the following strategies :

Period 1 : Both players play C respectively.
Period 2 : Both players play G if first period outcome is (C,C), else play b.

Note that these strategies yield an equilibrium in the second period regardless of the first period outcome. Furthermore, given the above specified strategies, no player want to deviate provided that $k \geq 2$.

2.2 Independent Private Monitoring Case.

We will illustrate that there is no way to enforce cooperation in the first period when private monitoring is independent. Suppose that first period action profile(a_1,a_2) is not observed. Rather, each player i observes a signal y_i{c,d} about her opponents's action. Suppose that:

$$\Pr(y_i=c|a_j) = 1-\varepsilon \quad \text{if } a_j=C$$
$$= \varepsilon \quad \text{if } a_j=D.$$

Monitoring is almost perfect if ε is small. However, strikingly, there is no way to support pure-strategy equilibrium (C, C) in the first period. The reason is that in the second period, player i will want to play G if and only if she assigns probability (1/(k+1)) or greater to j playing G. Consider strategies that call for each player to play C in the first period, and then G in second period if and only if y_i=C. These strategies are irrelevant since if player i plays C then she assigns probability $1-\varepsilon$ to j observing C, and hence to j playing G. That is, regardless of the signal she observes, she will want to play G, and so she won't follow the strategies. Essentially, the information structure says that both players' second period information is independent. So long as player I cooperates in the first period, she will assign high probability to j observing a good signal regardless of her own signal, Eventually, she prefers to just keep cooperating. This argument says that there is no way to coordinate on the punishment equilibrium in period 2 following a bad outcome, and on the good equilibrium otherwise when private monitoring is independent.

2.3 Correlated Private Monitoring

In this section, we will show that if the private signals are highly, but not perfectly correlated, then it will be possible to support cooperation in the first period by coordinating on different second period play depending on the signals received. That is, the coordination problem is lessened when the private signals are correlated rather independent. Let's consider following extreme case of perfect correlation. Suppose that $y_i=y_j=y\in\{c,d\}$, where

$$\begin{aligned} Pr(y=c|(a_i=a_j)) &= 1-\varepsilon \quad \text{if } a_i=a_j=C \\ &= \varepsilon \quad \text{otherwise} \end{aligned}$$

This is a case of an imperfect public monitoring. Consider strategies that call for each player to play C in the first period, and to play G in the second period if and only if they observe c. This strategy yields an equilibrium in the second period. To show this, check the first period incentives:

$$\text{Play C} \Rightarrow 1+(1-\varepsilon)k+\varepsilon$$
$$\text{Play D} \Rightarrow 2+\varepsilon k+1-\varepsilon$$

If $k\geq 1+1/(1-2\varepsilon)$, then it is optimal to follow the specified strategy. This result reverts to the perfect monitoring case when ε is small. If signals y_i and y_j are highly correlated(but not perfectly correlated), then it is possible to enforce cooperation in the first period by coordinating on different second period play depending on the signals.

2.4 Mixed Strategies

In the case of independent private monitoring, we know that there is no way to enforce cooperation in the first period. What happens to the beliefs of players if they play the mixed strategies. To see this, let's return to the independent signals set-up from above. consider the following strategies.

Period 1 : Play C, D with probabilities α, $1-\alpha$.
Period 2 : play G if and only if a_i=C and y_i=c.

In the second period of coordination game, player i will want to play G if and only if she assigns 1/(k+1) or greater to j playing G. She assigns this probability belief by using Bayes rules, that is, conditioning on what she knows, her action a_i and her signal y_i.

$$Pr(j \text{ will play } G|\ a_i,y_i) = Pr(y_j=c|a_i)\times Pr(a_j=C|y_i)$$

For example, $Pr(j \text{ will play } G|C,c) = Pr(y_j=c|C)\times Pr(a_j=C|c) = Pr(y_j=c|C)\times[Pr(a_j=C\cap y_i=c)/(Pr(y_i=c))]=[Pr(y_j=c|C)\times\{Pr(y_i=c|a_j=C)\times Pr(a_j=C)\}]/\{Pr(y_i=c|a_j=C)\times Pr(a_j=C)+Pr(y_i=c|a_j=D)\times Pr(a_j=D)$. So $Pr(j \text{ will play } G|C,c)=(1-\varepsilon)\times(1-\varepsilon)\alpha/\{\alpha(1-\varepsilon)+(1-\alpha)\varepsilon\}$.

This implies that player I will be willing to follow the prescribed strategy in the second period if ε goes to zero. Just now consider player i's incentives in the first period. If she plays C in the first period. her first period payoff is $\alpha+(1-\alpha)(-1)$. In the second period the probability of the state of coordinating playing G will be $Pr(j \text{ will play } G|a_i=C,y_i=c)\times Pr(y_j=c)$. So the probability of the state of that is $(1-\varepsilon)^2\alpha$. His expected payoffs in the second period are $\alpha(1-\varepsilon)^2(k-1)+1$. The total payoff of playing C is $\alpha+(1-\alpha)(-1)+\alpha(1-\varepsilon)^2(k-1)+1$. On the other hand the expected payoff of player i's being playing D is $2\alpha+1$. We can make her just indifferent between C and D by setting $\alpha=1/(k-1)(1-\varepsilon)^2$.

Ⅲ. Different Approaches to Private Monitoring

Above examples illustrates some of the issues arising with private monitoring within two period of play. In this section we will consider more general approaches to the repeated play, and what has been shown. Most of the existing literature focus on infinite repeated play of the prisoner's dilemma (with $\iota \geq g \geq 0$):

Player I \ Player II	C	D
C	1,1	$-\iota$, 1+g
D	1+g, $-\iota$	0,0

3.1 Failure of Grim Trigger Strategies

Compte (2002) showed that grim trigger strategies fail to enforce cooperation if private signals are conditionally independent of the action profiles that was played. His point is that when player i observes a signal that is supposed to trigger entry to the punishment phase, she will be very reluctant to initiate punishment unless she assigns high probability to j having already entered the punishment phase. On the other hand, she would not have been willing to play C to begin with if she assigns high probability to j having entered the punishment phase. This implies that with conditionally independent private signals the only pure strategy equilibrium is to play D in every period. The approaches below will try to get around this problem in different ways.

3.2 Sekiguchi's initial randomization

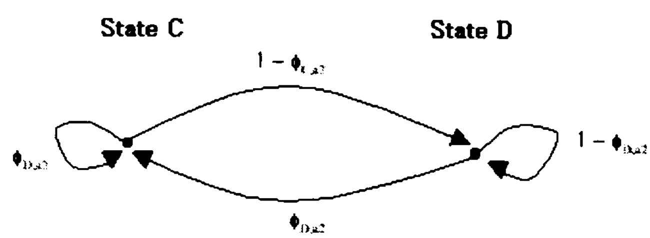
Sekiguchi (1997) was the first one who constructed an equilibrium that can approximately obtain the cooperative payoff (1,1) in the prisoner's dilemma game under almost perfect private monitoring. His work initiated the rapidly growing literature: Bhaskar and Obara (2002) extended Sekiguchi's framework to support any point Pareto dominating (0,0); Piccione (2002) introduced completely different machine game approach to support essentially the same area under almost perfect monitoring.

Sekiguchi (197) requires that at the beginning of the game, both players randomize between playing grim trigger strategies and playing defect in every period. He shows that this is an equilibrium for intermediate discount factor provided that monitoring is sufficiently accurate. Sekiguchi has to tackle two issues to verify the equilibrium. Suppose that player i has been cooperating and has always received good signals. Will she still want to cooperate? It turns out that she is highly patient and monitoring is sufficiently accurate. The second question is that she will want to defect as soon as a bad signal is received. If she receives the bad signal at the beginning period, she can conclude that her opponent started with the always defect strategy, so she is happy to defect. On the other hand, if she receives the bad signal only after some time, the she is confused that either (1) the signal is erroneous, or (2) her opponent received a bad signal in the previous period and has just entered punishment phase. These events have equal probability. Sekiguchi manages to show that if discount factor is not too high, player i will be willing to experiment by defecting. Thus he obtains cooperation under almost perfect private monitoring.

3.3 Piccione's Ongoing Randomization

Piccione (2002) introduced a general technique to construct uncoordinated punishment in infinitely repeated games. Note that if we had more than two stages presented in section 2, constructing strategies to support cooperation would be fairly complex since computing one's beliefs about the opponent's private history becomes more demanding after a few stages. Piccione introduced an ingenious idea that get around this problem. He considers the repeated prisoner's dilemma with private monitoring, and constructed an equilibrium strategy represented by a machine with countably many states. Piccione constructed a machine in such a way that each player is always indifferent between C and D no matter which state the opponent is in. Piccione's construction was substantially simplified independently by Ely and Valimali (2002). They showed that construction similar to Piccion's can be obtained by just two states.

Figure 1 Ely and Valimaki's two-state machine



To see how this might been done, let's consider the perfect monitoring version of the repeated prisoner's dilemma. Following Ely and Valimaki, consider strategies as follows.

Player I plays C in the first period, and in any subsequent period t, plays a mixed strategy depends only on the outcome at t-1. Let $\varphi^{i}_{ai,aj}$ denote the probability that she plays C at t on the outcome profile (a_i,a_j) st t-1. We can think of this strategy as a two-state machine, as is shown in Figure 1.

Ely and Valimaki claim that for any pair $(v_1,v_2)\in V = (0,1]^2$, there are strategies that achieve the pair. To see this, fix values V_C^i, $V_D^I \in(0,1]$, with V_C^i, V_D^I. We now show that we can find probabilities pair φ^j, such that (1) in any period where j plays C, player i is indifferent between playing C and D respectively and obtains continuation payoff V_C^i, and (2) in any period where player j plays D, player i is indifferent between playing C and D respectively and obtains continuation payoff V_D^i. This requires finding four probabilities $\varphi^j \in[0,1]$ to satisfy the four equations:

$$V_C^I = (1-\delta)+\delta[\varphi^j_{cc}V_C^i+(1-\varphi^j_{cc})V_D^i]$$
$$V_C^I = (1-\delta)(1+g)+\delta[\varphi^j_{cd}V_C^i+(1-\varphi^j_{cd})V_D^i]$$
$$V_D^I = (1-\delta)(-\iota)+\delta[\varphi^j_{dc}V_C^i+(1-\varphi^j_{dc})V_D^i]$$
$$V_D^I = (1-\delta)0+\delta[\varphi^j_{dd}V_C^i+(1-\varphi^j_{dd})V_D^i]$$

These can be solved if δ is near enough 1. So we can obtain any payoff in the unit square in an equilibrium where players are indifferent between actions after every history. Essentially the same construction continues to work within the structure of imperfect private monitoring structure provided that this signal is nearly perfect. Piccione (2002) and Ely and Valimaki (2002) prove the Folk Theorem that individually feasible rational payoff set can be obtained. To do this they have a preliminary phase of finite length and support different payoffs in this preliminary phases with different continuation payoffs from the unit square.

VI. Some Comments

We will conclude this review paper with a little comments on some literature not reviewed in the previous sections. Matsushima (2002) shows that even if signals are conditionally independent and not nearly perfect the Folk Theorem holds in the Prisoner's dilemma game. He uses the Piccione-Ely-Valimaki approach. However their approach is hard to apply to general games without some modifications. There is also a potential criticism of this approach in the sense that the mixed strategies that they constructed seem quite fragile. In particular, Players should use different mixtures depending on their private history of play even though they always anticipate the same continuation payoffs. Even small payoff perturbations might upset this delicate indifference. Prior to Sekkguchi's initiative work on private monitoring in repeated play, There are works on private monitoring with communication in repeated games (for example, Compte (1998) and Kandori and Masushima (1998)). They allow the players to communicate through cheap talk following each period. Given that the players condition on the public announcements of their private histories, they prove the elegant extentions of the Fudenberg, Levine and Maskin (1994)'s Folk theorem.

Mailath and Morris (2002) also explore what might happen if there is private monitoring which is almost public monitoring.[1] Their idea is to start by considering a public monitoring technology and looking at some perfect public equilibrium under that technology. They then ask whether such a perfect public equilibrium is robust to the introduction of a small amount of private monitoring, which is almost public.

1) As in Fudenberg, Levine, and Maskin (1994), a public monitoring technology has a set of possible signals Y and a probability distribution over those signals depending of the action profiles, $\rho(y|a)$. Assume that this distribution has full support. With private monitoring, each player i observes a signal $y_i \in Y$, and there is a probability distribution $\pi(\mathbf{y}|a)$ where $\mathbf{y}$ is a signal profile. The notion of almost public monitoring by Mailath and Morris is that the private monitoring is ε-close to public monitoring technology: that is. for any action profile a, and any $y \in Y$, $|\pi(y,..,y|a) - \rho(y|a)| < \varepsilon$.

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