A Thesis for the Degree of M.E.

On T_λ-Continuous Function

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June, 1985

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이를 教育學碩士學位 論文으로 提出함.

濟州大學校 敎育大學院 數學敎育專攻

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1985 年 6月 日

姜大植의 碩士學位 論文을 認准함.

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1985 年 6月 日

감 사 의 글

이 논문이 완성되기 까지 연구에 바쁘신 가운데도 자상한 마음으로 친절하게 지도하여 주신 현진오교수님께 감사드리며, 아울러 지도와 편달을 아끼지 않으신 송석준교수님과 수학과 여러 교수님께 심심한 사의를 표합니다.

그리고 그동안 저에게 사랑과 격려를 하여주신 주위의 많은 분들께 또한 감사를 드립니다.

1985 년 6 월 일

강 대 식

Contents

1. Introduction

Weaker than Continuous functions have been a subject of interest in general topology since 1959 when Stallings, in [7], introduced the concepts of connectivity maps and almost continuous functions. Recent investigations can be seen in [1], [2], [3], [4] [5]. In the paper [5], the authors introduced three new types of non – continuous functions which have a close relationship with the separation axioms and continuous functions.

In this paper, we have some properties of Ti – continuous functions and some topological properties of them.

2. T_i - Continuous functions

Definition 2.1 ([5]) Let $(Y, \mathcal{J})$ be a topological space and let U be an open cover of $(Y, \mathcal{J})$. The cover U is said to be a T_2 - open cover of $(Y, \mathcal{J})$ provided if $u \in U$, then the interior of $Y - u$ is not empty.

The cover U is said to be a T_3 - open cover of $(Y, \mathcal{J})$ provided if $u \in U$, then there are open sets W_1 and W_2 such that $W_1 \subset \overline{W}_1 \subset W_2 \subset Y - u$.

Definition 2.2 ([5]) Let $(X, \mathcal{J}_1)$ and $(Y, \mathcal{J}_2)$ be topological spaces. A function $f: (X, \mathcal{J}_1) \rightarrow (Y, \mathcal{J}_2)$ is said to be T_1 - continuous (T_2 - continuous) (T_3 - continuous) provided if U is an open cover (T_2 - open cover) (T_3 - open cover) of $(Y, \mathcal{J}_2)$, then there exists an open cover V of $(X, \mathcal{J}_1)$ such that if $v \in V$, then there is a $u \in U$ such that $f(v) \subset u$.

3. On T_i – Continuous functions and separation axioms

Theorem 3.1 If $f : (X, \mathcal{J}_1) \to (Y, \mathcal{J}_2)$ and $g : (Y, \mathcal{J}_2) \to (Z, \mathcal{J}_3)$ are T_1 – continuous, then $gof : (X, \mathcal{J}_1) \to (Z, \mathcal{J}_3)$ is also T_1 – continuous.

Proof . Since g is T_1 – continuous, for any open cover W of $(Z, \mathcal{J}_3)$, there exists an open cover v of $(Y, \mathcal{J}_2)$ such that if $v \in V$, then there is a $w \in W$ such that $g(v) \subset W$. ——— (1)
Also, f is T_1 – continuous, for the given open cover V of $(Y, \mathcal{J}_2)$, there exists an open cover U of $(X, \mathcal{J}_1)$ such that if $u \in U$, then there is a $v' \in V$ such that $f(u) \subset v'$. ——— (2)
Hence, for any open cover W of $(Z, \mathcal{J}_3)$, there exists an open cover U of $(X, \mathcal{J}_1)$ such that if $u \in U$, there is a $w' \in W$ such that $(gof)(u) = g(f(u)) \subset g(v') \subset w'$ by (1) and (2).
Therefore $gof : (X, \mathcal{J}_1) \to (Z, \mathcal{J}_3)$ is also T_1 – continuous.

Corollary 3.1 (1) If $f : (X, \mathcal{J}_1) \to (Y, \mathcal{J}_2)$ is T_1 – continuous and $g : (Y, \mathcal{J}_2) \to (Z, \mathcal{J}_3)$ is T_2 – continuous, then $gof : (X, \mathcal{J}_1) \to (Z, \mathcal{J}_3)$ is also T_2 – continuous.

Proof . Since g is T_2 – continuous, for any T_2 – open cover W of $(Z, \mathcal{J}_3)$, there exists an open cover V of $(Y, \mathcal{J}_2)$ such that if $v \in V$, then there exists a $w \in W$ such that $g(v) \subset W$—(1)
Also, since f is T_1 – continuous, for the given open cover V of

$(Y,\mathcal{J}_2)$, there exists an open cover U of $(X,\mathcal{J}_1)$ such that if $u \in U$, then there is a $v' \in V$ such that $f(u) \subset v'$. —— (2)
Hence, for any T_2 – open cover W of $(Z,\mathcal{J}_3)$, there exists an open cover U of $(X,\mathcal{J}_1)$ such that if $u \in U$, there is a $w' \in W$ such that $(gof)(u) = g(f(u)) \subset g(v') \subset w'$ by (1) and (2).
Therefore, $gof : (X,\mathcal{J}_1) \rightarrow (Z,\mathcal{J}_3)$ is also T_2 – continuous.

Corollary 3.1 (2) If $f : (X,\mathcal{J}_1) \rightarrow (Y,\mathcal{J}_2)$ is T_1 – continuous and $g: (Y,\mathcal{J}_2) \rightarrow (Z,\mathcal{J}_3)$ is T_3 – continuous. then $gof: (X,\mathcal{J}_1) \rightarrow (Z,\mathcal{J}_3)$ is also T_3 – continuous.

Proof . Since g is T_3 – continuous, for any T_3 – open cover W of $(Z,\mathcal{J}_3)$, there exists an open cover V of $(Y,\mathcal{J}_2)$ such that if $v \in V$, then there exists a $w \in W$ such that $g(v) \subset W$. —— (1)
Also, since f is T_1 – continuous, for the given open cover V of $(Y,\mathcal{J}_2)$, there exists an open cover U of $(X,\mathcal{J}_1)$ such that if $u \in U$, then there is a $v' \in V$ such that $f(u) \subset v'$.—(2)
Hence, for any T_3 – open cover W of $(Z,\mathcal{J}_3)$, there exists an open cover U of $(X,\mathcal{J}_1)$ such that if $u \in U$, there is a $w' \in W$ such that $(gof)(u) = g(f(u)) \subset g(v') \subset w'$ by (1) and (2).
Therefore $gof : (X,\mathcal{J}_1) \rightarrow (Z,\mathcal{J}_3)$ is also T_3 – continuous.

Theorem 3.2 Let $(Y,\mathcal{J}_2)$ be a T_1 – space, then $f: (X,\mathcal{J}_1) \rightarrow (Y,\mathcal{J}_2)$ is T_1 – continuous if and only if f is continuous.

Proof.($\Rightarrow$) It is proved in [5].

($\Leftarrow$) Let U be an open cover of $(Y, \mathcal{J}_2)$,

then $\bigcup_{\alpha \in \mathscr{A}} U\alpha = Y$ for $u_\alpha \in U$ and $f^{-1}(u_\alpha)$ is open in X since f is continuous.

Then $V = \{f^{-1}(u_\alpha) \mid \alpha \in \mathscr{A}\}$ is an open cover of $(X, \mathcal{J}_1)$ since $\bigcup f^{-1}(u_\alpha) = f^{-1}(\bigcup u_\alpha) = f^{-1}(Y) = X$.

And if $v \in V$, then $v = f^{-1}(u_\beta)$ for some β.

Hence there exists $u_\beta \in U$ such that $f(v) = f(f^{-1}(u_\beta)) \subset u_\beta$.

Therefore f is T_1 − continuous.

Corollary 3.2 (1) Let $(Y, \mathcal{J}_2)$ be a T_2 − space. Then $f : (X, \mathcal{J}_1) \rightarrow (Y, \mathcal{J}_2)$ is T_2 − continuous if and only if f is continuous.

Proof. ($\Rightarrow$) It is proved in [5].

($\Leftarrow$) Let U be a T_2 − open cover of $(Y, \mathcal{J}_2)$,

then $\bigcup_{\alpha \in \mathscr{A}} u_\alpha = Y$ for $u_\alpha \in U$ and $f^{-1}(u_\alpha)$ is open in X since f is continuous.

Then $V = \{f^{-1}(u_\alpha) \mid \alpha \in \mathscr{A}\}$ is an open cover of $(X, \mathcal{J}_1)$ since $\bigcup f^{-1}(u_\alpha) = f^{-1}(\bigcup u_\alpha) = f^{-1}(Y) = X$.

And if $v \in V$, then $v = f^{-1}(u_\beta)$ for some β.

Hence there exists $u_\beta \in U$ such that $f(v) = f(f^{-1}(u_\beta)) \subset u_\beta$. Therefore f is T_2 − continuous.

Corollary 3.2 (2) Let $(Y, \mathcal{J}_2)$ be a T_3 – space. Then $f : (X, \mathcal{J}_1) \to (Y, \mathcal{J}_2)$ is T_3 – continuous if and only if f is continuous.

Proof. ($\Rightarrow$) It is proved in [5]

($\Leftarrow$) Let U be a T_3 – open cover of $(Y, \mathcal{J}_2)$,

then $\bigcup_{\alpha \in \mathscr{A}} u_\alpha = Y$ for $u_\alpha \in U$ and $f^{-1}(u_\alpha)$ is open in X since f is continuous.

Then $V = \{ f^{-1}(u_\alpha) \mid \alpha \in \mathscr{A} \}$ is an open cover of $(X, \mathcal{J}_1)$ since

$\bigcup f^{-1}(u_\alpha) = f^{-1} (\bigcup u_\alpha) = f^{-1}(Y) = X$.

And if $v \in V$, then $v = f^{-1}(u_\beta)$ for some β.

Hence there exists $u_\beta \in U$ such that

$f(v) = f(f^{-1}(u_\beta)) \subset u_\beta$.

Therefore f in T_3 – continuous.

Theorem 3.3 If $f : (X, \mathcal{J}_1) \to (Y, \mathcal{J}_2)$ is a T_1 – continuous and $A \subset X$, then the restriction.

$f / A : (A, \mathcal{J}_1 / A) \to (Y, \mathcal{J}_2)$ is also T_1 – continuous.

Proof. Since f is T_1 – continuous, for any open cover V of $(Y, \mathcal{J}_2)$, there exists an open cover U of $(X, \mathcal{J}_1)$ such that if $u \in U$, then there is a $v \in V$ such that $f(u) \subset v$.

since $A \subset X$, $U_A = \{ u_\alpha \cap A \mid u_\alpha \in U \}$ is an open cover of A with respect to U.

Hence for any $u_\alpha \cap A \in U_A$, $u_\alpha \cap A \subset u_\alpha$ and there exists $v_\beta \in V$ such that $f / A\, (u_\alpha \cap A) \subset f(u_\alpha) \subset v_\beta$. Hence for any open cover

V of $(Y, \mathcal{J}_2)$

there is an open cover U_A of $(A, \mathcal{J}_1/A)$

such that if $u \cap A \in U_A$, then there is a $v \in V$ such that

$f(u \cap A) \subset v$.

Therefore $f/A : (A, \mathcal{J}_1/A) \to (Y, \mathcal{J}_2)$ is also T_1 - continuous.

Lemma 3.4 Let $(X, \mathcal{J}_1)$ be a topoloyical space and let U be an open cover of $(X, \mathcal{J}_1)$.

If U is a T_3 - open cover of $(X, \mathcal{J}_1)$ and $A \subset X$ then $U_A = \{A \cap u \mid u \in U\}$ is also a T_3 - open cover of $(A, \mathcal{J}_1/A)$.

Proof . Since U is a T_3 - open cover of $(X, \mathcal{J}_1)$ for any $u \in U$, there exist open sets W_1 and W_2 in $(X, \mathcal{J}_1)$ such that $W_1 \subset \overline{W_1} \subset W_2 \subset Y - u$.

Then $W_1 \cap A$, $W_2 \cap A \in \mathcal{J}_1/A$ and

$W_1 \cap A \subset \overline{W_1} \cap A \subset W_2 \cap A \subset (Y-u) \cap A$.

But $\overline{W_1} \cap A$ equals to the closure of $W_1 \cap A$ in $\mathcal{J}_1/A$ and $(Y-u) \cap A = A - (u \cap A)$.

Hence for any $u \cap A \in U_A$, there exist

$W_1 \cap A$, $W_2 \cap A \in \mathcal{J}_1/A$ such that

$W_1 \cap A \subset cl_A (W_1 \cap A) \subset W_2 \cap A \subset A - (u \cap A)$.

Therefore $U_A = \{A \cap u \mid u \in U\}$ is also a T_3 - open cover of $(A, \mathcal{J}_1/A)$.

Theorem 3.5 If $f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is a T_3 - continuous and $A \subset X$, then the restriction $f / A : (A, \mathcal{T}_1 / A) \to (Y, \mathcal{T}_2)$ is also T_3 - continuous.

Proof . Since f is T_3 - continuous, for any T_3 - open cover V of $(Y, \mathcal{T}_2)$, there exists a T_3 - open cover U of $(X, \mathcal{T}_1)$ such that if $u \in U$, then there is a $v \in V$ such that $f(u) \subset V$ since $A \subset X$, $U_A = \{ u_\alpha \cap A \mid u_\alpha \in U \}$ is a T_3 - open cover of A with respect to U by alove Lemma.

Hence for any $u_\alpha \cap A \in U_A$, $u_\alpha \cap A \subset u_\alpha$ and there exists $v_\beta \in V$ such that $f / A (u_\alpha \cap A) \subset f(u_\alpha) \subset v_\beta$.

Hence for any T_3 - open cover V of $(Y, \mathcal{T}_2)$ there is a T_3 - open cover U_A of $(A, \mathcal{T}_1 / A)$ such that if $u \cap A \in U_A$, then there is a $v \in V$ such that $f(u \cap A) \subset v$.

Therefore $f / A : (A, \mathcal{T}_1 / A) \to (Y, \mathcal{T}_2)$ is also T_3 - continuous.

Therrem 3.6 Let $X = A \cup B$, where A and B are closed in $(X, \mathcal{T}_1)$. Let $f : (A, \mathcal{T}_1 / A) \to (Y, \mathcal{T}_2)$ and $g : (B, \mathcal{T}_1 / B) \to (Y, \mathcal{T}_2)$ be T_1 - continuous.

If $f(x) = g(x)$ for every $x \in A \cap B$, then f and g combine to give a T_1 - continuous function $h : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ defined by setting $h(x) = f(x)$ if $x \in A$, and $h(x) = g(x)$ if $x \in B$.

Proof. Let V be an open cover of $(Y, \mathcal{J}_2)$. Then there exist open covers U_A of $(A, \mathcal{J}_1/A)$ and U_B of $(B, \mathcal{J}_1/B)$ such that if $u_A \in U_A$, then there is $v \in V$ such that $f(u_A) \subset V$ ang if $u_B \in U_B$, then there is $v' \in V$ such that $f(u_B) \subset v'$.

If we put $U = \{ u \in \mathcal{J}_1 \mid u \cap A \in U_A \} \cup \{ u \in \mathcal{J}_1 \mid u \cap B \in U_B \}$, we have that U is an open cover of $(X, \mathcal{J}_1)$.

Since $u_A = u \cap A$ for some $u \in \mathcal{J}_1$ and $u_B = u \cap B$ for some $u \in \mathcal{J}_1$, we have that if $u \in U$, $u = u \cap X = u \cap (A \cup B)$ $= (u \cap A) \cup (u \cap B) = u_A \cup u_B$,

then there exists $v'' (= v' \cup v) \in V$ such that $f(u) = f(u_A \cup u_B) = f(u_A) \cup f(u_B) \subset v \cup v' = v''$.

Hence $h : (X, \mathcal{J}_1) \to (Y, \mathcal{J}_2)$ is T_1 - continuous.

Corollarg 3.6 (1) Let $X = A \cup B$, where A and B are closed in $(X, \mathcal{J}_1)$. Let $f : (A, \mathcal{J}_1/A) \to (Y, \mathcal{J}_2)$ and $g : (B, \mathcal{J}_1/B) \to (Y, \mathcal{J}_2)$ be T_2 - continuous.

If $f(x) = g(x)$ for every $x \in A \cap B$, then f and g combine to give a T_2 - continuous funotion $h : (X, \mathcal{J}_1) \to (Y, \mathcal{J}_2)$ defined by setting $h(x) = g(x)$ if $x \in A$, and $h(x) = g(x)$ if $x \in B$.

Proof . Let V be a T_2 - open cover of $(Y, \mathcal{J}_2)$.

Then there exist open cover U_A of $(A, \mathcal{J}_1/A)$ and U_B of $(B, \mathcal{J}_1/B)$ such that if $u_A \in U_A$, then there is $v \in V$ such that

$f(u_A) \subset v$ and if $u_B \in U_B$, then there is $v' \in V$ such that $f(u_B) \subset v'$.

If we put $U = \{u \in \mathcal{J}_1 \mid u \cap A \in U_A\} \cup \{u \in \mathcal{J}_1 \mid u \cap B \in U_B\}$, we have that U is an open cover of $(X, \mathcal{J}_1)$.

Since $u_A = u \cap A$ for some $u \in \mathcal{J}_1$ and $u_B = u \cap B$ for some $u \in \mathcal{J}_1$, we have that if $u \in U$, $u = u \cap X = u \cap (A \cup B) = (u \cap A) \cup (u \cap B) = u_A \cup u_B$,

then there exists $v''(= v' \cup v) \in V$ such that $f(u) = f(u_A \cup u_B) = f(u_A) \cup f(u_B) \subset v \cup v' = v''$.

Hence $h : (X, \mathcal{J}_1) \to (Y, \mathcal{J}_2)$ is T_2 - continuous.

Corollary 3.6 (2) Let $X = A \cup B$, where A and B are closed in $(X, \mathcal{J}_1)$, Let $f : (A, \mathcal{J}_1/A) \to (Y, \mathcal{J}_2)$ and $g : (B, \mathcal{J}_1/B) \to (Y, \mathcal{J}_2)$ be T_3 - continuous. If $f(x) = y(x)$ for every $x \in A \cap B$, then f and g combine to give a T_3 - continuous function $h : (X, \mathcal{J}_1) \to (Y, \mathcal{J}_2)$ defined by setting $h(x) = f(x)$ if $x \in A$, and $h(x) = g(x)$ if $x \in B$.

Proof. Let V be a T_3 - open cover of $(Y, \mathcal{J}_2)$. Then there exist open cover U_A of $(A, \mathcal{J}_1/A)$ and U_B of $(B, \mathcal{J}_1/B)$ such that if $u_A \in U_A$, then there is $v \in V$ such that $f(U_A) \subset v$ and if $u_B \in U_B$, then there is $v' \in V$ such that $f(u_B) \subset v'$.
If we put $U = \{u \in \mathcal{J}_1 \mid u \cap A \in U_A\} \cup \{u \in \mathcal{J}_1 \mid u \cap B \in U_B\}$ we have that U is an open cover of $(X, \mathcal{J}_1)$. Since $u_A = u \cap A$ for some $u \in \mathcal{J}_1$ and $u_B = u \cap B$ for some $u \in \mathcal{J}_1$, we have that if $u \in U$, $u = u \cap X = u \cap (A \cup B) = (u \cap A) \cup (u \cap B) = u_A \cup u_B$, then there exists $v''(= v' \cup v) \in V$ such that $f(u) = f(u_A \cup u_B) = f(u_A) \cup f(u_B) \subset v \cup v' = v''$.
Hence $h : (X, \mathcal{J}_1) \to (Y, \mathcal{J}_2)$ is T_3 - continuous.

4. Some Toplogical Properties on T_i – continuous function.

Theorem 4.1 If $f:(X,\mathcal{J}_1) \to (Y,\mathcal{J}_2)$ is T_1 – continuous and onto and $(X,\mathcal{J}_1)$ is Lindelöf, then $(Y,\mathcal{J}_2)$ is Lindelöf.

Proof. Let U be an open cover of $(Y,\mathcal{J}_2)$. Since f is T_1 – continuous, there is an open cover V of $(X,\mathcal{J}_1)$ such that if $v \in V$, then there is a $u \in U$ such that $f(v) \subset u$.
Since $(X,\mathcal{J}_1)$ is Lindelöf, there is a countable subcover $\{v_1, v_2, v_3, \cdots\}$ of V which covers $(X,\mathcal{J}_1)$.
If i is a positive integer ($i = 1,2,3,\cdots\cdots$),
let u_i be an element of U such that $f(v_i) \subset u_i$. Since f is onto, $\{u_1, u_2, \cdots\cdots\}$ covers $(Y,\mathcal{J}_2)$ and hence, $(Y,\mathcal{J}_2)$ is Lindelöf.

Corollary 4.1 (1) If $f:(X,\mathcal{J}_1) \to (Y,\mathcal{J}_2)$ is T_1 – continuous and onto and $(X,\mathcal{J}_1)$ is compact, then $(Y,\mathcal{J}_2)$ is compact.

Proof. It is proved in [5].

Lemma 4.2

(1) The continuous image of a compact set is compact.

(2) The Lindelöf property is invariant under continuous surj–

ections.

Proof. See ([6], P 224 1.4 Theorem, P 175 6.6 Theorem)

Corollary 4.2 (1) Let $(X,\mathcal{J}_1)$ be a compact and $(Y,\mathcal{J}_2)$ is T_2 – space. If f: $(X,\mathcal{J}_1) \rightarrow (Y,\mathcal{J}_2)$ is T_2 – continuous and onto, then $(Y,\mathcal{J}_2)$ is compact.

Proof. Corollary 3.1 (1) shows that f is continuous. And by Lemma 4.2 (1), $(Y,\mathcal{J}_2)$ is compact.

Corollary 4.2 (2) Let $(X,\mathcal{J}_1)$ be a Lindelöf and $(Y,\mathcal{J}_2)$ is T_2 – space

If f : $(X,\mathcal{J}_1) \rightarrow (Y,\mathcal{J}_2)$ is T_2 – continuous and onto, then $(Y,\mathcal{J}_2)$ is Lindelöf.

Proof . Corollary 3.1 (1) shows that f is continuous. And by Lemma 4.2(2) , $(Y,\mathcal{J}_2)$ is Lindelöf.

Corollary 4.2 (3) Let $(X,\mathcal{J}_1)$ be a compact and $(Y,\mathcal{J}_2)$ is T_3 – space

If f : $(X,\mathcal{J}_1) \rightarrow (Y,\mathcal{J}_2)$ is T_3 – continuous and onto, then $(Y,\mathcal{J}_2)$ is compact.

Proof . Corollary 3.1 (2) shows that f is continuous and by Lemma 4.2 (1), $(Y,\mathcal{J}_2)$ is compact.

Corollary 4.2 (4) Let $(X,\mathcal{J}_1)$ be a Lindelöf and $(Y,\mathcal{J}_2)$ is

T_3 − space.

If f : $(X, \mathcal{T}_1)$ → $(Y, \mathcal{T}_2)$ is T_3 − continuous and onto, then $(Y, \mathcal{T}_2)$ is Lindelöf.

Proof . Corollarg 3.1 (2) shows that f is continuous and by Lemma 4.2(2) , $(Y, \mathcal{T}_2)$ is Lindelöf.

Theorem 4.3 If f : $(X, \mathcal{T}_1)$ → $(Y, \mathcal{T}_2)$ is T_1 − continuous and onto and $(X, \mathcal{T}_1)$ is connected, then $(Y, \mathcal{T}_2)$ is connected

Proof . Suppose $(Y, \mathcal{T}_2)$ is not connected . Then $Y = A \cup B$ where $A \neq \varnothing$, $B \neq \varnothing$, $A, B \in \mathcal{T}_2$, and $A \cap B = \varnothing$.

Then $U = \{A, B\}$ is an open cover of $(Y, \mathcal{T}_2)$ and since f is T_1− continuous, there is an open cover V of $(X, \mathcal{T}_1)$ such that if $v \in V$ then there is a $u \in U$ such that $f(v) \subset u$.

Let $M = \cup\{v \in V$ and $f(v) \subset A\}$ and let $N = \cup\{v \in V$ and $f(v) \subset B\}$. Since f is onto, M and N are non-empty. Since $A \cap B = \varnothing$, it follows that $M \cap N = \varnothing$. Clearly M and N are in $\mathcal{T}_1$ and since V is an open cover of X, $X = M \cup N$.

But this is impossible since $(X, \mathcal{T}_1)$ is connected.

Thus $(Y, \mathcal{T}_2)$ is connected.

Corollary 4.3 (1) If f : $(X, \mathcal{T}_1)$ → $(Y, \mathcal{T}_2)$ is T_2 − continuous and onto and $(X, \mathcal{T}_1)$ is connected, then $(Y, \mathcal{T}_2)$ is connected.

Proof . Suppose $(Y, \mathcal{T}_2)$ is not connected. Then $Y = A \cup B$ where

$A \neq \emptyset$, $B \neq \emptyset$, $A, B \in \mathcal{J}_2$, and $A \cap B = \emptyset$

Then $U = \{A, B\}$ is a T_2 – open cover of $(Y, \mathcal{J}_2)$ and since f is T_2 – continuous, there is an open cover V of $(X, \mathcal{J}_1)$ such that if $v \in V$ then there is a $u \in U$ such that $f(v) \subset u$

Let $M = \cup \{v \in V$ and $f(v) \subset A\}$ and let $N = \cup \{v \in V$ and $f(v) \subset B\}$. Since f is onto, M and N are non – empty. Since $A \cap B = \emptyset$, it follows that $M \cap N = \emptyset$. Clearly M and N are in $\mathcal{J}_1$ and since V is an open cover of X, $X = M \cup N$. But this is impossible since $(X, \mathcal{J}_1)$ is connected.

Thus $(Y, \mathcal{J}_2)$ is connected.

Corollary 4.3 (2) If $f : (X, \mathcal{J}_1) \rightarrow (Y, \mathcal{J}_2)$ is T_3 – continuous and onto and $(X, \mathcal{J}_1)$ is connected, then $(Y, \mathcal{J}_2)$ is connected.

Proof. It is similar to the proof of corollary 4.3 (1).

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< 國 文 抄 錄 >

分離空間을 갖는 連續函數에 關하여

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連續函數보다 弱한 條件을 갖는 函數들에 關한 研究는 1959 年 Stallings 이 發表한 類의 連續函數에 關한 論文 以後에 重要한 研究 對象이 되어 왔다. ([7])

최근에는 Gauld 를 비롯한 여러 外國 位相數學 研究者들과 황석근등의 國內 位相數學者들에 依해서도 研究되고 있다.

本 論文은 이들의 研究들을 參照하고 特히 Gentry와 Hoyle 이 定義한 T_i-連續函數를 보다 깊이 研究하여 몇가지 位相的性質을 얻게 되었다. (3 장)

또한 이 性質들을 Compact 및 Connected 와 結合하여 T_i - 連續函數의 不變性을 研究하였다. (4 장)