Theoretical study on the optical detection of magnetophonon resonance in semiconductors in tilted magnetic fields

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Ⅰ. Introduction

Magnetophonon resonance (MPR) arises from an electron resonant scattering due to absorption and emission of phonons when the energy separation between two of Landau levels is equal to a phonon energy. Since the pioneer work by Gurevich and Firsov [1], this effect has been extensively studied as a powerful spectroscopic tool for investigating transport behavior of electrons in bulk[2,3] and low-dimensional semiconductor systems[4-10]. The MPR enables us to obtain information on band structure parameters, such as the effective mass and the energy levels, and on the electron-phonon interaction. The vast majority of work on the MPR has been done on the transport properties of semiconductors, usually the magnetoresistance, which

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inevitably involves a complicated average of scattering processes. The oscillations in the magnetoresistance are the results of a combination of scattering and broadening processes that can lead to a quite complicated dependence of the resonance amplitudes on doping, sample structure, carrier concentration, and temperature. However, the MPR can also be observed directly through a study of the electron cyclotron resonance (CR) linewidth and effective mass, i.e., the so-called optically detected MPR (ODMPR), as was demonstrated in three-dimensional (3D) semiconductor systems of GaAs by Hai et al. [11] and in two-dimensional (2D) semiconductor systems of $GaAs/Al_xGa_{1-x}As$ heterojunctions by Barnes et al. [12] The ODMPR allows one to make quantitative measurements of the scattering strength for specific Landau levels and yields direct information on the nature of the electron-phonon interaction in semiconductors.

The purpose of the present work is to obtain a general form of frequency-dependent magnetoconductivity, by using the Mori-type projection operator technique presented by one of the present authors[13], and to present the explicit expressions of the lineshape function and the linewidth closely related to the optically detected magnetophonon resonances due to various transitions including intraband, intervalley, and interband transition in bulk semiconductors, which are expressed in two different ways for a weak coupling and an arbitrary and/or strong one.

The present paper is organized as follows : In Sec. II, we will describe the model of the system. The frequency-dependent magnetoconductivity for the system modeled in the previous section is evaluated in Sec. III. The conductivity is closely related to the lineshape function due to the collision process. In Sec. IV, the general expressions of lineshape functions for a weak electron-phonon interaction and an arbitrary and/or strong one are obtained by using the Mori-type projection operator technique. The explicit expressions of the lineshape function and the linewidth for a weak coupling and an arbitrary and/or strong one are given in Sec. V, which is related to the optically detected magnetophonon resonances due to various transitions including intraband, intervalley, and interband transition in bulk semiconductors. Our results are summarized in the last section.

II. Model of the system

Consider a system of many non-interacting electrons N_e in interaction with phonons, initially in equilibrium with a temperature T. Then, in the presence of a static magnetic field tilted with an angle of θ from the z axis chosen to be parallel to the principal axis of an ellipsoidal energy surface, $\boldsymbol{B} = B(\sin\theta, 0, \cos\theta)$, the time-independent Hamiltonian H of the system can be written as

$$\begin{aligned} H &= H_e + V + H_p \\ &= \sum_{\lambda s}\sum_{\lambda' s'} \langle \lambda s | (h_e + v) | \lambda' s' \rangle a^{+}_{\lambda s} a_{\lambda' s'} + H_p, \end{aligned} \tag{2.1}$$

$$h_e = \frac{1}{2}(\boldsymbol{p} + e\boldsymbol{A}) \begin{pmatrix} \frac{1}{m_t} & 0 & 0 \\ 0 & \frac{1}{m_t} & 0 \\ 0 & 0 & \frac{1}{m_l} \end{pmatrix} (\boldsymbol{p} + e\boldsymbol{A}), \tag{2.2}$$

$$v = \sum_{\boldsymbol{q}} [\gamma^{+}_{\boldsymbol{q}} b^{+}_{\boldsymbol{q}} + \gamma_{\boldsymbol{q}} b_{\boldsymbol{q}}], \tag{2.3}$$

$$H_p = \sum_{\boldsymbol{q}} \hbar\omega_{\boldsymbol{q}} (b^{+}_{\boldsymbol{q}} b_{\boldsymbol{q}} + \frac{1}{2}), \tag{2.4}$$

$$\gamma_{\boldsymbol{q}} = C_{\boldsymbol{q}} \exp(i\boldsymbol{q} \cdot \boldsymbol{r}) \tag{2.5}$$

where $|\lambda s\rangle$ means the electron state in the $s$band or valley : λ denotes the Landau state (N, $\boldsymbol{k}$), $N(=0,1,2,\cdots)$ and $s$are the Landau-level index and band or valley index, respectively. $a^{+}_{\lambda s}(a_{\lambda s})$ is the creation (annihilation) operator for an electron with momentum $\boldsymbol{p}$, $\boldsymbol{A}$ denotes the vector potential, m_t and m_l represent the transverse and longitudinal mass components of the ellipsoidal energy surface of the conduction band, respectively, $b^{+}_{\boldsymbol{q}}(b_{\boldsymbol{q}})$ is the creation (annihilation) operator for a phonon with momentum $\hbar\boldsymbol{q}$ and energy $\hbar\omega_{\boldsymbol{q}}$, $C_{\boldsymbol{q}}$ is the interaction operator, and $\boldsymbol{r}$ is the position vector of an electron. By taking into account the Landau guage of vector potential $\boldsymbol{A} = B(-y\cos\theta, 0, y\sin\theta)$ the one-electron normalized eigenfuncitons ($\langle \boldsymbol{r} | \lambda s\rangle$) and eigenvalues (E^{s}_{λ}) in the s band or valley are given, respectively, by

$$\langle \boldsymbol{r} | \lambda s\rangle \equiv \langle \boldsymbol{r} | N, k_x, k_z, s\rangle = U^{s}_{0}(\boldsymbol{r}) F^{s}_{\lambda}(\boldsymbol{r}), \tag{2.6}$$

$$E_\lambda^s = E_N^s(k_x, k_z) = (N+1/2)\hbar\omega_s + \hbar^2(\mathbf{k}\cdot \mathbf{B}/B)^2/2m_B^s \tag{2.7}$$

where k_x and k_z are, respectively, the wavevector component of the electron in the x and z direction, $\omega_s(=eB/m_s^*)$ is the cyclotron frequency in the s band or valley, and m_B^s means the effective mass in the magnetic field direction. Here

$$\frac{1}{m_s^2} = \frac{\cos^2\theta}{m_t^2} + \frac{\sin^2\theta}{m_t m_l}, \tag{2.8}$$

$$m_B^s = m_l\cos^2\theta + m_t\sin^2\theta \tag{2.9}$$

Also, in Eq. (2.6), $U_0^s(\mathbf{r})$ denotes the Bloch function of the sband or valley at $\mathbf{k}=0$ and $F_\lambda^s(\mathbf{r})$ means the envelope function given by

$$F_\lambda^s(\mathbf{r}) = \frac{1}{L_{xL_z}}\phi_N(y-y_\lambda^s)\exp(ik_x x + ik_z z) \tag{2.10}$$

with

$$y_\lambda^s = \frac{\hbar}{eBm_B^s}[m_t k_z \sin\theta - m_l k_x\cos\theta], \tag{2.11}$$

where $\phi_N(y)$ in Eq. (2.10) are the eigenfunctions of the simple harmonic oscillator and L_x and L_z are, respectively, the x- and z-directional normalization lengths. We assume that the Bloch function $U_0^s(\mathbf{r})$ and the envelope function $F_\lambda^s(\mathbf{r})$ are, respectively, normalized in the crystal as

$$\int_C U_0^{s'*}(\mathbf{r})U_0^s(\mathbf{r})d^3r = \delta_{s,s'}, \tag{2.12}$$

$$\int_\Omega F_{\lambda'}^{s'}*(\mathbf{r})F_\lambda^s(\mathbf{r})d^3r = \delta_{\lambda,\lambda'} = \delta_{N,N'}\delta_{k_x,k_x'}\delta_{k_z,k_z'} \tag{2.13}$$

where C is the volume of the unit cell and $\Omega(=L_xL_yL_z)$ is the crystal volume in the real space.

If we apply the energy eigenvalues of Eq. (2.7) to many-valley model and multiband model, we can get the energy eigenvalues

$$E_{\lambda}^{s} = E_{N}^{s}(k_x, k_z) = (N+1/2)\hbar\omega_s + \hbar^2(\boldsymbol{k}\cdot \boldsymbol{B}/B)^2/2m_B^s, \tag{2.14}$$

$$E_{\lambda'}^{s'} = E_{N'}^{s'}(k_x', k_z') = (N'+1/2)\hbar\omega_{s'} + \hbar^2(\boldsymbol{k}'\cdot \boldsymbol{B}/B)^2/2m_B^s \tag{2.15}$$

for s and s' valleys, respectively, and

$$E_{\lambda}^{c} = E_{N}^{c}(\boldsymbol{k}) = E_g + (N+1/2)\hbar\omega_c + \hbar^2(\boldsymbol{k}\cdot \boldsymbol{B}/B)^2/2m_B^s, \tag{2.16}$$

$$E_{\lambda'}^{V} = E_{N'}^{V}(k_x', k_z') = -(N'+1/2)\hbar\omega_V - \hbar^2(\boldsymbol{k}'\cdot \boldsymbol{B}/B)^2/2m_B^V \tag{2.17}$$

for the conduction and valence bands, respectively, where E_g is the energy gap, $\omega_C = eB/m_C^s$ and $\omega_V = eB/m_V$. C and V in the superscript or subscript indicate the conduction and valence bands, respectively. We see from Eqs. (2.16) and (2.17) that in the presence of the magnetic field, the conduction and valence bands separated at $\boldsymbol{k}=0$ by the direct-band-gap E_g are split into Landau subbands specified by the Landau level indices $N(N') = 0, 1, 2, \cdots$, in which the energies of the single-electron in the conduction and valence bands are quantized in the y-direction and quasi-continous in the x-z plane. It is interesting to note from Eqs. (2.14)-(2.17) that the energy separation of two Landau levels in the s valley are different from that of two Landau levels in the s' valley, as in the conduction band and the valence band, and that the slopes of the energy band in two different valleys are same each other whereas those of the energy band in two different bands are different. The minimum of the lowest subband of the conduction band occurs at the energy $\hbar\omega_C/2$above the energy band minimum in zero fields, and the maximum of the highest subband in the valence band occurs at the energy $\hbar\omega_V/2$ below the valence band maximum in zero fields, thus the band gap is given by $E_g + \hbar(\omega_C + \omega_V)/2$. In addition, it is to be noted that MPR or ODMPR effects of interest to us can be observed in zero-gap materials, such as HgTe, because these effects do not take place in the case where the band gap is larger than the longitudinal optical phonon energy.

Ⅲ. Theory of Optically Detected Magnetophonon Resonance

When a linearly polarized electromagnetic wave of amplitude E and frequency ω given by

$$E_x = 0,\ E_y = E\cos\omega t,\ E_z = 0 \tag{3.1}$$

is applied along the line tilted with an angle of θ from the z axis in the zx-plane, the absorption power delivered to the system is given for the Faraday configuration ($E \perp B$) as [14]

$$P = (E^2/4)\ \mathrm{Re}\,\sigma_{yy}(\overline{\omega}) + \sigma_{yy}(-\overline{\omega})] \tag{3.2}$$

where Re means "the real part of", $\overline{\omega} = \omega - i\delta$ ($\delta \rightarrow 0^+$) and $\sigma_{yy}(\overline{\omega})$ (or $\sigma_{yy}(-\overline{\omega})$) is the complex optical conductivity corresponding to the right(or left)-circularly polarized wave, which can be expressed in the Kubo formalism [15] as

$$\begin{aligned} \sigma_{yy}(\overline{\omega}) &= \Omega^{-1}\int_0^\infty dt\exp(-i\overline{\omega}t)\int_0^\beta d\beta_1 T_R[\rho_{eq}J_y(-i\hbar\beta_1|H)J_y(t|H)] \\ &= \lim_{u_y \to 0}\frac{\partial}{\partial u_y}\Omega^{-1}\int_0^\infty dt\exp(-i\overline{\omega}t)T_R[\rho_{eq}(\widetilde{H})J_y(t|H)] \end{aligned}, \tag{3.3}$$

where Ω represents the volume of the system defined in Eq. (2.13), T_R means the many-body trace, J_y is the y-component of the total current operator in the many-body formalism, and $J_y(t|H)$ is the time-dependent total current operator in the Heisenberg representation.

In order to express Eq. (3.3) in the single electron representation for an electron-phonon system we assume that the statistical operator $\rho_{eq}(\widetilde{H})$ in Eq. (3.3) is factorized as [13]

$$\rho_{eq}(\widetilde{H}) \approx \rho_{ph}(H_p)\rho[\sum_i h_e^{(i)} - \vec{u}\cdot\vec{J}] \tag{3.4}$$

and the many--body trace T_R is reduced to $tr \cdot T_R^{(ph)}$. Here $\rho_{ph}(H_p) \equiv \exp(-\beta H_p)/T_R^{(ph)}[\exp(-\beta H_p)]$ and the symbols tr and $T_R^{(ph)}$ mean the single electron trace and the many--body trace over the phonon scatterings, respectively. Then, the frequency--dependent conductivity formula, Eq. (3.3), can be expressed in terms of the single electron trace as

$$\sigma_{yy}(\overline{\omega}) = \Omega^{-1}\int_0^{\infty} dt\exp(-i\overline{\omega}t)\langle tr \lim_{u_y\to 0}(\frac{\partial f}{\partial u_y})j_y(t|h_e+v+H_p)\rangle_{ph}, \tag{3.5}$$

where $\langle\cdots\rangle_{ph}$ denotes the average over the phonon scatterings and f is the modified Fermi--Dirac operator given by

$$f\equiv[\exp\beta(h_e+\vec{u}\cdot\vec{j}-\zeta)+1]^{-1}. \tag{3.6}$$

In order to rewrite Eq. (3.5) in more convenient form, we represent the interaction term in the phonon average as

$$tr\left\{\lim_{u_y\to 0}(\frac{\partial f}{\partial u_y})j_y(t|h_e+v+H_p)\right\} = \sum_{\lambda s,\lambda' s'}\frac{1}{2\pi i}\oint_c dzf(z)\langle\lambda s|R_z j_y R_z|\lambda' s'\rangle \times \langle\lambda' s'|dj_{y(t}h_e+v+H_p)|\lambda s\rangle, \tag{3.7}$$

where we have used

$$\lim_{u_y\to 0}\frac{\partial}{\partial u_y}(z-h_e-\vec{u}\cdot\vec{j})^{-1} = R_z j_y R_z \tag{3.8}$$

with $R_z=(z-h_e)^{-1}$ and $f(z)$ is defined by $f(z)=[\exp\beta(z-\zeta)+1]^{-1}$. The $|\lambda s\rangle$ in Eq. (3.7) denotes the single electron state given in Eq. (2.6).

Then, by considering Eqs. (3.5) and (3.7) the frequency-dependent conductivity for the right-circularly polarized wave is reduced to

$$\sigma_{yy}(\omega) = \Omega^{-1}\sum_{\lambda s,\lambda' s'}\langle\langle\lambda s|Y_y|\lambda' s'\rangle\langle\lambda' s'|\widetilde{j_y(\overline{\omega})}|\lambda s\rangle\rangle_{ph}, \tag{3.9}$$

where $\widetilde{j}_y(\overline{\omega})$ is the Fourier-Laplace transform (FLT) of $j_y(t|h_T)$ defined by

$$\widetilde{j}_y(\overline{\omega})\equiv FLT j_y(t|h_T) = \int_0^{\infty} dt\exp(-i\overline{\omega}t)j_{y(t}|h_{T)} \tag{3.10}$$

with $h_T\equiv h_e+v+H_p$ and

$$\langle\lambda s|Y_y|\lambda' s'\rangle = \frac{f(E_\lambda^s)-f(E_{\lambda'}^{s'})}{E_\lambda^s-E_{\lambda'}^{s'}}\langle\lambda s|j_y|\lambda' s'\rangle. \tag{3.11}$$

Here E_λ^s is the energy eigenvalue of the Hamiltonian h_e given in Eq. (2.7).

Then, the frequency--dependent conductivity formula for the right-circularly polarized wave can be rewritten from Eqs. (3.9) and (3.11) as

$$\sigma_{yy}(\overline{\omega}) = \Omega^{-1} \sum_{\lambda s, \lambda' s'} \frac{f(E_\lambda^s) - f(E_{\lambda' s'})}{E_\lambda^s - E_{\lambda' s'}} \langle \lambda s | j_y | \lambda' s' \rangle \ll \lambda' s' | \widetilde{j}_y(\overline{\omega}) | \lambda s \gg_{ph}, \qquad (3.12)$$

It should be noted that the conductivity formula for the left-circularly polarized wave given in Eq. (3.2) can be obtained from replacing $\overline{\omega}$ in Eq. (3.12) by $-\overline{\omega}$. The central problem of Eq. (3.12) is the evaluation of the configuration over the phonon fields. Especially, the main task is then to give a suitable expansion method for the operators $\ll \lambda' s' | \widetilde{j}_y(\overline{\omega}) | \lambda s \gg_{ph}$ in Eq. (3.12), which will be outlined in the following section.

Ⅲ. Lineshape Function

In order to obtain the lineshape function we will present two representations using the Mori-type projection operator technique [13]; a closed--form representation and a continued--fraction form representation.

A. The closed form

For the calculation of $\ll \lambda' s' | \widetilde{j}_y(\overline{\omega}) | \lambda s \gg_{ph}$ in Eq. (3.12), we define the projection operators P_0 and P_0' for the states $|\lambda' s'\rangle$ and $|\lambda s\rangle$ as

$$P_0 X = (X_{fi}/j_{yfi}) j_y,$$

$$P_0' = 1 - P_0, \qquad (4.2)$$

where $X_{fi} \equiv \ll \lambda' s' | X | \lambda s \gg_{ph}$ for any operator X.

Following Mori [16], we separate $j_y(t|h_T)$ into the projective and vertical components with respect to the j_y-axis as

$$j_y(t|h_T) = P_0 j_y(t|h_T) + P_0' j_y(t|h_T)$$

$$= Z_{0fi}(t|h_T) j_y + \int_0^t dt_1 Z_{0fi}(t_1|h_T) f_1'(t-t_1|h_T)$$

where

$$Z_{0fi}(t\,|h_T) \equiv j_{yfi}(t\,|\,h_T)/j_{yfi}, \tag{4.4}$$

$$f_1'(t\,|h_T) \equiv \exp(iL_1 t/\hbar) f_1', \tag{4.5}$$

$$f_1' \equiv iL_1 j_y/\hbar, \tag{4.6}$$

$$L_1 \equiv P_0' L_T. \tag{4.7}$$

$$L_T \equiv L_e + L_v + L_{ph}. \tag{4.8}$$

Here L_e, L_v, and L_{ph} are Liouville operators corresponding to the single electron Hamiltonian h_e, the scattering potential v, and the phonon Hamiltonian, respectively.

In order to obtain $\tilde{j}_{yfi}(\overline{\omega})$ or $\tilde{Z}_{0fi}(\overline{\omega})$, we differentiate Eq. (4.4) as

$$\frac{d}{dt} Z_{0fi}(t\,|h_T) = i\omega_{0fi} Z_{0fi}(t\,|h_T) + \int_0^t dt_1 \Delta_{0fi}(t-t_1\,|h_T) Z_{0fi}(t_1\,|\,h_T) \tag{4.9a}$$

$$= i\omega_{0fi} Z_{0fi}(t\,|\,h_T) + \int_0^t dt_1 Z_{1fi}(t-t_1\,|\,h_T)\Delta_{0fi} Z_{0fi}(t_1\,|h_T). \tag{4.9b}$$

Here

$$\omega_{0fi} \equiv (L_T j_y/\hbar)_{fi}/j_{yfi} = (E_f - E_i)/\hbar, \tag{4.10}$$

$$\Delta_{0fi}(t\,|\,h_T) \equiv f_{1fi}(t\,|11h_T)/j_{yfi} \equiv Z_{1fi}(t\,|\,h_T)_{0fi}\Delta_{0fi}, \tag{4.11}$$

$$f_{1(t}|\,h_T) = iL_T f_1'(t\,|\,h_T)/\hbar, \tag{4.12}$$

$$Z_{1fi}(t\,|\,h_T) \equiv f_{1fi}(t\,|h_T)/f_{1fi}, \tag{4.13}$$

$$\Delta_{0fi} \equiv f_{1fi}/j_{yfi}, \tag{4.14}$$

where E_i and E_f correspond to E^s_λ and $E^{s'}_{\lambda'}$ given by Eqs. (2.14) and (2.15), respectively, and we have used $v_{ff} - v_{ii} = 0$ in Eq. (4.10).

Then, the FLT of Eqs. (4.9a) and (4.9b) leads to

$$\tilde{Z}_{0fi}(\overline{\omega}) \equiv \tilde{j}_{yfi}(\overline{\omega})/j_{yfi} = [i\overline{\omega} - i\omega_{0fi} + \Sigma_{0fi}(\overline{\omega})]^{-1}. \tag{4.15}$$

Here $\Sigma_{0fi}(\overline{\omega})$, often called the frequency--dependent self--energy operator, is defined as

$$\Sigma_{0fi}(\overline{\omega}) = -\tilde{\Delta}_{0fi}(\overline{\omega}) \tag{4.16a}$$

$$= -\tilde{Z}_{1fi}(\overline{\omega})\,\tilde{\Delta}_{0fi}, \tag{4.16b}$$

where $\tilde{\Delta}_{0fi}(\overline{\omega})$ and $\tilde{Z}_{1fi}(\overline{\omega})$ are the Fourier-Laplace transform of Eqs. (4.11) and (4.13), respectively. Considering Eqs. (4.5)-(4.8), (4.11), (4.12) and (4.16a), and taking into account the relation $P_0(L_e+L_{ph})G_{0ph}P_0'X = [(L_e+L_{ph})G_{0ph}P_0'X]_{fi} = 0$ we obtain

$$\Sigma_{0fi}(\overline{\omega})\,\hbar j_{yfi})^{-1} \langle \sum_{N=1}^{\infty}[(L_v G_{0ph}P_0')^{NL_T} j_y]_{fi} \rangle_{ph} \quad ,$$

where $G_{0ph} = (\hbar\overline{\omega} - L_e - L_{ph})^{-1}$ and we have used the relation $(A-B)^{-1} = A^{-1}\sum_{m=0}^{\infty}(BA^{-1})^m$ for any operators A and B. Now the self-energy operator $\Sigma_{0fi}(\overline{\omega})$ has been expanded with respect to L_v corresponding to the scattering potential. Eq. (4.17) is the general formula for the frequency-dependent self-energy operator given in a closed expansion form for electron-phonon systems, which is applicable to the weak coupling case since we have taken the relation $(A-B)^{-1} = A^{-1}\sum_{m=0}^{\infty}(BA^{-1})^m$. Eq. (4.17) is identical with Choi et al' result [17] obtained by Argyres-Sigel's projection operator method [18] in the cyclotron resonance transition problem. According to their result, L_T in Eq. (4.17) has been replaced by $L_v$under the assumption that $P_0 L_e X = 0$ for any operator X. Note that this assumption is not always satisfied and it can give us different results of the lineshape function. For example, for intraband transition, the lineshape function obtained to the second order scattering strength gives us same results irrespective of whether the assumption is used or not while for interband transition the lineshape function obtained to the second order scattering strength gives us different results. The detailed thing will be discussed in Sec. V.

B. The continued-fraction form

In order to obtain $\tilde{Z}_{1fi}(\bar{\omega})$ of Eq. (4.16b) we define the projection operators P_1 and P_1' as

$$P_1X = (X_{fi}/f_{1fi})f_1, \tag{4.18}$$

$$P_1' = 1 - P_1. \tag{4.19}$$

By utilizing these operators we separate $f_{1(t}|h_T)$ into the projective and vertical components with respect to the f_1-axis as

$$\begin{aligned} f_{1(t}|h_{T)} &= P_{1f_{1(t}}|h_T) + P_1'f_1(t|h_T) \\ &= Z_{1fi}(t|h_T)f_1 + \int_0^t Z_{1fi}(t_1|h_T)f_2'(t-t_1|h_T)dt_1 \end{aligned}, \tag{4.20}$$

where

$$f_2'(t|h_T) \equiv \exp(iL_2t/\hbar)f_2', \tag{4.21}$$

$$\dot{f_2} \equiv iL_2f_1/\hbar, \tag{4.22}$$

$$L_2 \equiv P_1'L_TP_0', \tag{4.23}$$

In order to obtain $\tilde{Z}_{1fi}(\bar{\omega})$, we differentiate Eq. (4.13) as

$$\begin{aligned} \frac{d}{dt}Z_{1fi}(t|h_T) &= i\omega_{1fi}Z_{1fi}(t|h_T) + \int_0^t dt_1\Delta_{1fi}(t-t_1 \quad h_T)Z_{1fi}(t_1|h_T \\ &= i\omega_{1fi}Z_{1fi}(t|h_T) + \int_0^t dt_1Z_{2fi}(t-t_1|h_T)\Delta_{1fi}Z_{1fi}(t_1|h_T) \end{aligned}, \tag{4.24}$$

where

$$\omega_{1fi} \equiv (L_TP_0'f_1/\hbar)_{fi}/f_{1fi}, \tag{4.25}$$

$$\Delta_{1fi}(t \quad h_T) \equiv f_{2fi}(t|h_{T)/f_{1fi}} \equiv Z_{2fi}(t|h_T)\Delta_{1fi}, \tag{4.26}$$

$$f_{2(t}|h_T) = iL_TP_0'f_2'(t|h_T)/\hbar, \tag{4.27}$$

$$Z_{2fi}(t|h_T) \equiv f_{2fi}(t)/f_{2fi}, \tag{4.28}$$

$$\Delta_{1fi} \equiv f_{2fi}/f_{1fi}. \tag{4.29}$$

Then the FLT of Eq. (4.24) leads to

$$\tilde{Z}_{1fi}(\overline{\omega}) \equiv \tilde{f}_{1fi}(E_l)/f_{1fi} = [i\overline{\omega} - i\omega_{1fi} + \tilde{\Sigma}_{1fi}(\overline{\omega})]^{-1}. \tag{4.30}$$

Here $\tilde{\Sigma}_{1fi}(\overline{\omega})$ is the first-order frequency--dependent self--energy operator in the continued--fraction forms given by

$$\tilde{\Sigma}_{1fi}(\overline{\omega}) = -\tilde{\Delta}_{1fi}(\overline{\omega}) \tag{4.31a}$$

$$= -\tilde{Z}_{2fi}(\overline{\omega})\Delta_{1fi} \tag{4.31b}$$

We now see that $\tilde{\Delta}_{1fi}(\overline{\omega})$, the FLT of Eq. (4.26), is given in a closed--form expansion as in Eq. (4.17) while $\tilde{Z}_{2fi}(\overline{\omega})$, the FLT of Eq. (4.28), can be given in a continued-fracton manner via the successive projection operators onto the f_2, f_3, f_4 $\cdots$ axes as follows.

In order to obtain the general form for $\tilde{Z}_{jfi}(\overline{\omega})$ we define the projection operators P_j and P_j' onto the f_j axis as

$$P_jX = (X_{fi}/f_{jfi})f_j, \tag{4.32}$$

$$P_j' = 1 - P_j. \tag{4.33}$$

Thus we have

$$\begin{aligned} f_{j(t}|h_{T)} &= iL_T\Pi_{m=0}^{j-2}P_m'f_j'(t\,|\,h_T)/\hbar = P_{jf_j}(t\,|\,h_T) + P_j'f_j(t\,|\,h_T) \\ &= Z_{jfi}(t\,|\,h_{T)f_j} + \int_0^t Z_{jfi}(t_1\,|\,h_T)f_{j+1}'(t-t_1\,|\,h_T)dt_1 \end{aligned} \tag{4.34}$$

where the notation $\Pi_{m=0}^{j-2}P_m'$ means $P_0'P_1'P_2'\cdots P_{j-2}'$ and

$$Z_{jfi}(t\,|h_T) = f_{jfi}(t\,|\,h_T)/f_{jfi}, \tag{4.35}$$

$$f_{j+1}'(t\,|h_T) \equiv \exp(iL_{j+1}t/\hbar)f_{j+1}', \tag{4.36}$$

$$f_{j+1}' \equiv iL_{j+1}f_{j}/\hbar, \tag{4.37}$$

$$L_{j+1} \equiv P_j' L_T \Pi_{m=0}^{j-1} P_m'. \tag{4.38}$$

Then the time derivative of $Z_{jfi}(t \mid h_T)$ leads to

$$\begin{aligned}\frac{d}{dt} Z_{jfi}(t \mid h_T) &= i\omega_{jfi} Z_{jfi}(t \mid h_T) + \int_0^t dt_1 \Delta_{jfi}(t-t_1 \mid h_T) Z_{jfi}(t_1 \mid h_T) \\ &= i\omega_{jfi} Z_{jfi}(t \mid h_T) + \int_0^t dt_1 Z_{j+1fi}(t-t_1 \mid h_T) \Delta_{jfi} Z_{jfi}(t_1 \mid h_T)\end{aligned} \quad ,(4.39)$$

where

$$\omega_{jfi} \equiv (L_T \Pi_{m=0}^{j-1} P_m' f_{jl} \hbar)_{fi} / f_{jfi}, \tag{4.40}$$

$$\Delta_{jfi}(t \mid h_T) \equiv f_{j+1fi}(t \mid h_T)/f_{jfi} \equiv Z_{j+1fi}(t \mid h_T)\Delta_{jfi}, \tag{4.41}$$

$$f_{j+1}(t \mid h_T) = iL_T \Pi_{m=0}^{j-1} P_m' f_{j+1}'(t \mid h_T)/\hbar, \tag{4.42}$$

$$\Delta_{jfi} \equiv f_{j+1fi}/f_{jfi} \quad .$$

The FLT of Eq. (4.39) leads to

$$\tilde{Z}_{jfi}(\bar{\omega}) \equiv \tilde{f}_{jfi}(\bar{\omega})/f_{jfi} = [i\bar{\omega} - i\omega_{jfi} - \tilde{\Delta}_{jfi}(\bar{\omega})]^{-1} \tag{4.44a}$$

$$= [i\bar{\omega} - i\omega_{jfi} - \tilde{Z}_{j+1fi}(\bar{\omega})\Delta_{jfi}]^{-1}, \quad (0 \le j \le \infty) \tag{4.44b}$$

where $\tilde{\Delta}_{jfi}(\bar{\omega})$ and $\tilde{Z}_{j+1fi}(\bar{\omega})$ are the FLT of Eqs. (4.41) and (4.43), respectively. Equation (4.44a) is given in a closed--form expansion in the j--th continued--fraction representation, as in Eqs. (4.16a) and (4.31a). By considering Eqs. (4.15), (4.16b), (4.30), (4.31b) and (4.44b), we obtain the general frequency--dependent self--energy operator given in a continued-fraction :

$$\begin{aligned}\Sigma_{0fi}(\bar{\omega}) &= \frac{-\Delta_{0fi}}{i\bar{\omega} - i\omega_{1fi} + \Sigma_{1fi}(\bar{\omega})} \\ &= \cfrac{-\Delta_{0fi}}{i\bar{\omega} - i\omega_{1fi} - \cfrac{\Delta_{1fi}}{i\bar{\omega} - i\omega_{2fi} - \cfrac{\Delta_{2fi}}{i\bar{\omega} - i\omega_{3fi} - \cfrac{\Delta_{3fi}}{i\bar{\omega} - i\omega_{4fi} - \ddots}}}},\end{aligned} \tag{4.45}$$

where Δ_{0fi}, Δ_{1fi}, $\cdots$ and ω_{1fi}, ω_{2fi}, $\cdots$ can be easily obtained from Eqs. (4.40) and (4.43). Note that $\Sigma_{1fi}(\overline{\omega})$ in Eq. (4.45) is given in two forms expressed by the infinite closed-form expansion of the finite continued fraction order and the infinite continued fraction representation. We see that Eq. (4.45) is applicable to the strong coupling case.

Considering Eqs. (3.12), (4.10), and (4.15) we can express the frequency--dependent magnetooptical conductivity tensor for the right-circularly polarized wave as

$$\sigma_{yy}(\overline{\omega}) = \frac{\hbar}{i\Omega}\sum_{f,i}\frac{f(E_f)-f(E_i)}{E_f-E_i}\frac{|j_{yfi}|^2}{\hbar\overline{\omega}-E_f+E_i-i\hbar\Sigma_{0fi}(\overline{\omega})}. \tag{4.46}$$

It should be noted that the $|i\rangle\ (\equiv|\lambda s\rangle)$and the $E_i\ (\equiv E^s_\lambda)$ are, respectively, the eigenstates and eigenvalues of h_e given in Eqs. (2.6) and (2.7). The lineshape function, $i\hbar\Sigma_{0fi}(\overline{\omega})$, results in the lifetime broadening, which is responsible for the spectral broadening of lineshape. Therefore, the real and imaginary part of $i\hbar\Sigma_{0fi}(\overline{\omega})$ defined by

$$i\hbar\Sigma_{0fi}(\overline{\omega}) \equiv \hbar\widetilde{\nabla}_{0fi}(\overline{\omega})+i\hbar\Gamma_{0fi}(\overline{\omega}) \tag{4.47}$$

are the lineshift and linewidth, respectively, for the transition arising from the resonant absorption of a single photon of frequency ω and of a single phonon of frequency ω_q between states $|i\rangle$ and $|f\rangle$. Real and imaginary parts of Eq. (4.47) are of basic interest and are related to the quantities measured experimentally. The real part provides the resonance shifting whereas the imaginary part gives directly the average value of the relaxation time, the inverse of which then measures the resonance broadening of the absorption spectrum. It shoud be noted that both of these quantites are given by a function of temperature, the strength and/or the tilt angle of magnetic field, the incident photon frequency, the difference in the effective mass between initial and final states of the intervalley or interband scattering by phonon, and the involved phonon energy. The self-energy results in the lifetime broadening, which is responsible for the spectral broadening of lineshapes. Therefore, the collision broadening effect by scattering are studied theoretically by examining the real part of the conductivity tensor.

The remarkable thing here is that Eq. (4.46) has different forms for various transitions including intraband, intervalley, and interband transitions. For intraband transition, the selection rule is given by $\langle \lambda' s' | j_y | \lambda s \rangle = j_{y\lambda+1\lambda} \delta_{s's} \delta_{\lambda'\lambda+1}$ in Eq. (4.46). E_i and E_f in Eq. (4.46) are replaced by $E_\lambda (\equiv E_\lambda^s)$and $E_{\lambda+1} (\equiv E^s_{\lambda+1})$, respectively. For intervalley transition, the selection rule is given by $\langle \lambda' s' | j_y | \lambda s \rangle = j_{ys's} \delta_{\lambda'\lambda}$ in Eq. (4.46). E_i and E_f in Eq. (4.46) are replaced by E_λ^s and $E_\lambda^{s'}$ given by Eqs. (2.14) and (2.15), respectively. Furthermore, for interband transition the selection rule is given by $\langle \lambda' s' | j_y | \lambda s \rangle = j_{ycv} \delta_{\lambda'\lambda}$ in Eq. (4.46). E_i and E_f in Eq. (4.46) are replaced by E_λ^c and E_λ^v given by Eqs. (2.16) and (2.17), respectively.

Ⅳ. Explicit Expression for the Lineshape Function

In this section we shall derive an explicit expression of the lineshape function for various transitions in the case of both weak coupling and strong coupling given in Eqs. (4.17) and (4.45), respectively. The central interest in the evaluation of Eqs. (4.17) and (4.45) is averaging over the phonon configurations.

A. Weak coupling case

For the second order of the scattering potential in Eq. (4.17) we obtain the lineshape functions for both intraband transition and intervalley and/or interband transition, respectively, as follows :

$$
\begin{aligned}
i\hbar\, \Sigma_{0\lambda+1\lambda}(\bar{\omega}) = & \sum_q (1+n_q)\Big[\sum_{\alpha(\neq\lambda+1)} \frac{(\gamma_q)_{\lambda+1\alpha}\{(\gamma_q^+)_{\alpha\lambda+1} - (\gamma_q^+)_{\alpha-1\lambda} j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}}{\hbar\bar{\omega} - E_\alpha + E_\lambda - \hbar\omega_q} \\
& + \sum_{\alpha(\neq\lambda)} \frac{\{(\gamma_q)_{\lambda\alpha} - (\gamma_q)_{\lambda+1\alpha+1} j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q^+)_{\alpha\lambda}}{\hbar\bar{\omega} - E_{\lambda+1} + E_\alpha + \hbar\omega_q} \Big] \\
& + \sum_q n_q \Big[\sum_{\alpha(\neq\lambda+1)} \frac{(\gamma_q^+)_{\lambda+1\alpha}\{(\gamma_q)_{\alpha\lambda+1} - (\gamma_q)_{\alpha-1\lambda} j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}}{\hbar\bar{\omega} - E_\alpha + E_\lambda + \hbar\omega_q} \\
& + \sum_{\alpha(\neq\lambda)} \frac{\{(\gamma_q^+)_{\lambda\alpha} - (\gamma_q^+)_{\lambda+1\alpha+1} j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q)_{\alpha\lambda}}{\hbar\bar{\omega} - E_{\lambda+1} + E_\alpha - \hbar\omega_q} \Big]
\end{aligned}
\tag{5.1a}
$$

$$i\hbar\ \Sigma_{0\lambda s'\lambda s}(\bar{\omega})=\sum_{q}(1+n_{q})\Big[\sum_{\alpha(\neq\lambda)}\frac{(\gamma_{q})_{\lambda s'\alpha s'}\{(\gamma_{q}^{+})_{\alpha s'\lambda s}-(\gamma_{q}^{+})_{\alpha s\lambda s}j_{\gamma\alpha s'\alpha s}/j_{\gamma\lambda s'\lambda s}\}}{\hbar\bar{\omega}-E_{\alpha}^{s'}+E_{\lambda}^{s}-\hbar\omega_{q}}$$
$$+\frac{\{(\gamma_{q})_{\lambda s\alpha s}-(\gamma_{q})_{\lambda s'\alpha s}j_{\gamma\alpha s'\alpha s}/j_{\gamma\lambda s'\lambda s}\}(\gamma_{q}^{+})_{\alpha\lambda}}{\hbar\bar{\omega}-E_{\lambda}^{s'}+E_{\alpha}^{s}+\hbar\omega_{q}}\Big]$$

$$+\sum_{q}\sum_{\alpha(\neq\lambda+1)}n_{q}\Big[\frac{(\gamma_{q}^{+})_{\lambda s'\alpha s'}\{(\gamma_{q})_{\alpha s'\lambda s'}-(\gamma_{q})_{\alpha s\lambda s}j_{\gamma\alpha s'\alpha s}/j_{\gamma\lambda s'\lambda s}\}}{\hbar\bar{\omega}-E_{\alpha}^{s'}+E_{\lambda}^{s}+\hbar\omega_{q}}$$
$$+\frac{\{(\gamma_{q}^{+})_{\lambda s\alpha s}-(\gamma_{q}^{+})_{\lambda s'\alpha s'}j_{\gamma\alpha s'\alpha s}/j_{\gamma\lambda s'\lambda s}\}(\gamma_{q})_{\alpha s\lambda s}}{\hbar\bar{\omega}-E_{\lambda}^{s'}+E_{\alpha}^{s}-\hbar\omega_{q}}\Big]$$

$$+\sum_{q}\sum_{\alpha(\neq\lambda)}(1+n_{q})\frac{\{(E_{\lambda}^{s'}-E_{\lambda}^{s})-(E_{\alpha}^{s'}-E_{\alpha}^{s})\}}{(\hbar\bar{\omega}-E_{\alpha}^{s'}+E_{\lambda}^{s})}$$
$$\Big[\frac{(\gamma_{q})_{\lambda s'\alpha s'}(\gamma_{q}^{+})_{\alpha s\lambda s}j_{\gamma\alpha s'\alpha s}/j_{\gamma\lambda s'\lambda s}}{\hbar\bar{\omega}-E_{\alpha}^{s'}+E_{\lambda}^{s}-\hbar\omega_{q}}+\frac{(\gamma_{q})_{\lambda s'\alpha s'}(\gamma_{q}^{+})_{\alpha s\lambda s}j_{\gamma\alpha s'\alpha s}/j_{\gamma\lambda s'\lambda s}}{\hbar\bar{\omega}-E_{\lambda}^{s'}+E_{\alpha}^{s}+\hbar\omega_{q}}\Big]$$

$$+\sum_{q}\sum_{\alpha(\neq\lambda)}n_{q}\frac{\{(E_{\lambda}^{s'}-E_{\lambda}^{s})-(E_{\alpha}^{s'}-E_{\alpha}^{s})\}}{(\hbar\bar{\omega}-E_{\alpha}^{s'}+E_{\lambda}^{s})}$$
$$\Big[\frac{(\gamma_{q}^{+})_{\lambda s'\alpha s'}(\gamma_{q})_{\alpha s\lambda s}j_{\gamma\alpha s'\alpha s}/j_{\gamma\lambda s'\lambda s}}{\hbar\bar{\omega}-E_{\alpha}^{s'}+E_{\lambda}^{s}+\hbar\omega_{q}}+\frac{(\gamma_{q}^{+})_{\lambda s'\alpha s'}(\gamma_{q})_{\alpha s\lambda s}j_{\gamma\alpha s'\alpha s}/j_{\gamma\lambda s'\lambda s}}{\hbar\bar{\omega}-E_{\lambda}^{s'}+E_{\alpha}^{s}-\hbar\omega_{q}}\Big]\tag{5.1b}$$

where $n_{\vec{q}}=[\exp(\beta\hbar\omega_{\vec{q}})-1]^{-1}$ is the phonon distribution function and $E_{\lambda}\equiv E_{\lambda}^{s}$ in Eq. (5.10). To obtain Eq. (5.1), we have used the relations

$$\sum_{\lambda}(P_0'X)_{\lambda\alpha}=\sum_{\lambda(\neq\alpha+1)}X_{\lambda\alpha},\tag{5.2}$$

$$\sum_{\lambda s}(P_0'X)_{\lambda s'\alpha s}=\sum_{\lambda(\neq\alpha)}X_{\lambda s'\alpha s},\tag{5.3}$$

$$\langle q\,|\,(G_{op}X)_{ij}\,|\,q'\rangle=\frac{\langle q\,|\,X_{ij}\,|\,q'\rangle}{\hbar\bar{\omega}-E_i+E_j-\langle q\,|\,H_p\,|\,q\rangle+\langle q'\,|\,H_p\,|\,q'\rangle},\tag{5.4}$$

$$G_{op}=(\hbar\bar{\omega}-L_e-L_p)^{-1}\tag{5.5}$$

for the phonon state

$$|\,q\rangle\equiv|\,n_{q1},n_{q2},n_{q3},\cdots,n_{qi},\cdots\rangle\tag{5.6}$$

and for any operator X. Here the matrix element is given with respect to both the electron states ($|i\rangle, |j\rangle$) and the phonon states ($|q\rangle, |q'\rangle$). Eq. (5.1a) is identical with Choi et al' result [17] obtained by Argyres-Sigel's projection operator method [18] and with Ryu et al.' result [19] obtained by Kawabata's projection operator method [20] in the cyclotron transition. Equation (5.1) is good for sufficiently weak scattering which neglects the many-body coherence effect. Note that for interband transition, the fifth and sixth terms on the right-hand side of Eq. (5.1b) appear because of $(E_\lambda^{s'} - E_\lambda^{s}) \neq (E_a^{s'} - E_a^{s})$ whereas for intervalley transition, the fifth and sixth terms on the right-hand side of Eq. (5.1b) do not exist because of $(E_\lambda^{s'} - E_\lambda^{s}) = (E_a^{s'} - E_a^{s})$. If the fifth and sixth terms on the right-hand side of Eq. (5.1b) are neglected, Eq. (5.1b) is identical with that of Choi et al. [21] and Yi et al. [14] obtained in the theory of interband magnetooptical transition.

B. Strong coupling case

By considering Eq. (4.45) given in a continued--fraction manner we can obtain the general formula in the strong coupling case. In order to obtain the lineshape function $i\hbar\ \Sigma_{0fi}(\overline{\omega})$, we must evaluate the quantities Δ_{0fi} and ω_{1fi} given in Eqs. (4.14) and (4.25), respectively :

$$\Delta_{0fi} \equiv f_{1fi}/j_{yfi} = -(L_T P_0' L_T j_k/\hbar^2)_{fi}/j_{yfi} \tag{5.7}$$

$$\omega_{1fi} \equiv (L_T P_0' f_1/\hbar)_{fi}/f_{1fi} = -(L_T P_0' L_T P_0' L_T j_k/\hbar^3)_{fi}/f_{1fi} \tag{5.8}$$

These quantities are contained in Eq. (4.45) and should be averaged over the phonon configurations. The results for various transition cases are given by

$$\langle q|\Delta_{0fi}|q\rangle = \begin{cases} -S_{p1}/\hbar^2 \ (\text{for intraband transition}) \\ -S_{p2}/\hbar^2 \ (\text{for intervalley and/or interband transition}) \end{cases}, \tag{5.9}$$

$$\langle q|\omega_{1fi}|q\rangle = \begin{cases} S_{p3}/\hbar S_{p1} \ (\text{for intraband transition}) \\ S_{p4}/\hbar S_{p2} \ (\text{for intervalley and/or interband transition}) \end{cases}, \tag{5.10}$$

where

$$S_{p1} = \sum_q (1+n_q)[\sum_{\alpha(\neq\lambda+1)} (\gamma_q)_{\lambda+1\alpha}\{(\gamma_q^+)_{\alpha\lambda+1} - (\gamma_q^+)_{\alpha-1\lambda} j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}$$
$$+ \sum_{\alpha(\neq\lambda)} \{(\gamma_q)_{\lambda\alpha} - (\gamma_q)_{\lambda+1\alpha+1} j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q^+)_{\alpha\lambda}]$$
$$+ \sum_q n_q[\sum_{\alpha(\neq\lambda+1)} (\gamma_q^+)_{\lambda+1\alpha}\{(\gamma_q)_{\alpha\lambda+1} - (\gamma_q)_{\alpha-1\lambda} j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}$$
$$+ \sum_{\alpha(\neq\lambda)} \{(\gamma_q^+)_{\lambda\alpha} - (\gamma_q^+)_{\lambda+1\alpha+1} j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q)_{\alpha\lambda}], \quad (5.11)$$

$$S_{p2} = \sum_q (1+n_q)[\sum_{\alpha(\neq\lambda+1)} (\gamma_q)_{\lambda s'\alpha s'}\{(\gamma_q^+)_{\alpha s'\lambda s'} - (\gamma_q^+)_{\alpha s\lambda s} j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}$$
$$+ \sum_{\alpha(\neq\lambda)} \{(\gamma_q)_{\lambda s\alpha s} - (\gamma_q)_{\lambda s'\alpha s'} j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q^+)_{\alpha s\lambda s}]$$
$$+ \sum_q n_q[\sum_{\alpha(\neq\lambda+1)} (\gamma_q^+)_{\lambda s'\alpha s}\{(\gamma_q)_{\alpha s'\lambda s'} - (\gamma_q)_{\alpha s\lambda s} j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}$$
$$+ \sum_{\alpha(\neq\lambda)} \{(\gamma_q^+)_{\lambda s\alpha s} - (\gamma_q^+)_{\lambda s'\alpha s'} j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q)_{\alpha s\lambda s}], \quad (5.12)$$

$$S_{p3} = \sum_q (1+n_q)[\sum_{\alpha(\neq\lambda+1)} (E_\alpha - E_\lambda + \hbar\omega_q)(\gamma_q)_{\lambda+1\alpha}\{(\gamma_q^+)_{\alpha\lambda+1} - (\gamma_q^+)_{\alpha-1\lambda} j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}$$
$$+ \sum_{\alpha(\neq\lambda)} (E_\alpha - E_\lambda - \hbar\omega_q)\{(\gamma_q)_{\lambda\alpha} - (\gamma_q)_{\lambda+1\alpha+1} j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q^+)_{\alpha\lambda}]$$

$$+ \sum_q n_q[\sum_{\alpha(\neq\lambda+1)} (E_{\lambda+1} - E_\alpha - \hbar\omega_q)(\gamma_q^+)_{\lambda+1\alpha}\{(\gamma_q)_{\alpha\lambda+1} - (\gamma_q)_{\alpha-1\lambda} j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}$$
$$+ \sum_{\alpha(\neq\lambda)} (E_{\lambda+1} - E_\alpha + \hbar\omega_q)\{(\gamma_q^+)_{\lambda\alpha} - (\gamma_q^+)_{\lambda+1\alpha+1} j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q)_{\alpha\lambda}] \quad (5.13)$$

$$S_{p4} = \sum_q (1+n_q)[\sum_{\alpha(\neq\lambda+1)} (E_\alpha^{s'} - E_\lambda^s + \hbar\omega_q)(\gamma_q)_{\lambda s'\alpha s'}\{(\gamma_q^+)_{\alpha s'\lambda s'} - (\gamma_q^+)_{\alpha s\lambda s} j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}$$
$$+ \sum_{\alpha(\neq\lambda)} (E_\lambda^{s'} - E_\alpha^s - \hbar\omega_q)\{(\gamma_q)_{\lambda s\alpha s} - (\gamma_q)_{\lambda s'\alpha s'} j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q^+)_{\alpha s\lambda s}]$$
$$+ \sum_q n_q[\sum_{\alpha(\neq\lambda+1)} (E_\alpha^{s'} - E_\lambda^s - \hbar\omega_q)(\gamma_q^+)_{\lambda s'\alpha s}\{(\gamma_q)_{\alpha s'\lambda s'} - (\gamma_q)_{\alpha s\lambda s} j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}$$
$$+ \sum_{\alpha(\neq\lambda)} (E_\lambda^{s'} - E_\alpha^s + \hbar\omega_q)\{(\gamma_q^+)_{\lambda s\alpha s} - (\gamma_q^+)_{\lambda s'\alpha s'} j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q)_{\alpha s\lambda s}]. \quad (5.14)$$

To derive Eqs. (5.11)-(5.14) we have used Eqs. (5.2)-(5.4) and utilized the fact that all terms including odd number of v disappear in the phonon average. Then taking into account Eqs. (4.45) and (5.9)-(5.14) we obtain the lineshape function for both intraband transition and intervalley and/or interband transition:

$$
\begin{aligned}
i\hbar\,\Sigma_{0\lambda+1\lambda}(\bar{\omega}) = \sum_q (1+n_q)\Big[&\sum_{\alpha(\neq\lambda+1)} \frac{(\gamma_q)_{\lambda+1\alpha}\{(\gamma_q^+)_{\alpha\lambda+1}-(\gamma_q^+)_{\alpha-1\lambda}j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}}{\hbar\bar{\omega}-E_\alpha+E_\lambda-\hbar\omega_q+\Xi_{1+}-i\hbar\,\Sigma_{1\lambda+1\lambda}(\bar{\omega})} \\
&+\sum_{\alpha(\neq\lambda)} \frac{\{(\gamma_q)_{\lambda\alpha}-(\gamma_q)_{\lambda+1\alpha+1}j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q^+)_{\alpha\lambda}}{\hbar\bar{\omega}-E_{\lambda+1}+E_\alpha+\hbar\omega_q+\Xi_{2-}-i\hbar\,\Sigma_{1\lambda+1\lambda}(\bar{\omega})}\Big] \\
+\sum_q n_q\Big[&\sum_{\alpha(\neq\lambda+1)} \frac{(\gamma_q^+)_{\lambda+1\alpha}\{(\gamma_q)_{\alpha\lambda+1}-(\gamma_q)_{\alpha-1\lambda}j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}}{\hbar\bar{\omega}-E_\alpha+E_\lambda+\hbar\omega_q+\Xi_{1-}-i\hbar\,\Sigma_{1\lambda+1\lambda}(\bar{\omega})} \\
&+\sum_{\alpha(\neq\lambda)} \frac{\{(\gamma_q^+)_{\lambda\alpha}-(\gamma_q^+)_{\lambda+1\alpha+1}j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q)_{\alpha\lambda}}{\hbar\bar{\omega}-E_{\lambda+1}+E_\alpha-\hbar\omega_q+\Xi_{2+}-i\hbar\,\Sigma_{1\lambda+1\lambda}(\bar{\omega})}\Big] \quad , \qquad (5.15)
\end{aligned}
$$

$$
\begin{aligned}
i\hbar\,\Sigma_{0\lambda s'\lambda s}(\bar{\omega}) = \sum_q (1+n_q)\Big[&\sum_{\alpha(\neq\lambda)} \frac{(\gamma_q)_{\lambda s'\alpha s'}\{(\gamma_q^+)_{\alpha s'\lambda s}-(\gamma_q^+)_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}}{\hbar\bar{\omega}-E_\alpha^{s'}+E_\lambda^{s}-\hbar\omega_q+\Xi_{3+}-i\hbar\,\Sigma_{1\lambda s'\lambda s}(\bar{\omega})} \\
&+\frac{\{(\gamma_q)_{\lambda s\alpha s}-(\gamma_q)_{\lambda s'\alpha s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q^+)_{\alpha\lambda}}{\hbar\bar{\omega}-E_\lambda^{s'}+E_\alpha^{s}+\hbar\omega_q+\Xi_{4-}-i\hbar\,\Sigma_{1\lambda s'\lambda s}(\bar{\omega})}\Big] \\
+\sum_q \sum_{\alpha(\neq\lambda+1)} n_q\Big[&\frac{(\gamma_q^+)_{\lambda s'\alpha s'}\{(\gamma_q)_{\alpha s'\lambda s'}-(\gamma_q)_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}}{\hbar\bar{\omega}-E_\alpha^{s'}+E_\lambda^{s}+\hbar\omega_q+\Xi_{3-}-i\hbar\,\Sigma_{1\lambda s'\lambda s}(\bar{\omega})} \\
&+\frac{\{(\gamma_q^+)_{\lambda s\alpha s}-(\gamma_q^+)_{\lambda s'\alpha s'}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q)_{\alpha s\lambda s}}{\hbar\bar{\omega}-E_\lambda^{s'}+E_\alpha^{s}-\hbar\omega_q+\Xi_{4+}-i\hbar\,\Sigma_{1\lambda s'\lambda s}(\bar{\omega})}\Big] \quad , \qquad (5.16)
\end{aligned}
$$

where $i\hbar\,\Sigma_{1\lambda+1\lambda}(\bar{\omega})$ and $i\hbar\,\Sigma_{1\lambda s'\lambda s}(\bar{\omega})$ in the denominator of Eqs. (5.15) and (5.16) are the high-order collision term given in Eq. (4.45) and

$$\Xi_{1\pm} = \{(E_\alpha - E_\lambda \pm \hbar\omega_q)S_{1p} - S_{3p}\}/S_{1p}, \qquad (5.17)$$

$$\Xi_{2\pm} = \{(E_{\lambda+1} - E_\alpha \pm \hbar\omega_q)S_{1p} - S_{3p}\}/S_{1p}, \qquad (5.18)$$

$$\Xi_{3\pm} = \{(E_\alpha^{s'} - E_\lambda^{s} \pm \hbar\omega_q)S_{2p} - S_{4p}\}/S_{2p}, \qquad (5.19)$$

$$\Xi_{4\pm} = \{(E_\lambda^{s'} - E_\alpha^{s} \pm \hbar\omega_q)S_{2p} - S_{4p}\}/S_{2p}. \qquad (5.20)$$

Eqs. (5.15) and (5.16) are the general formula of lineshape function for the strongly interacting electron-phonon scattering case. If the high order collision terms ($i\hbar\,\Sigma_{1\lambda+1\lambda}(\bar{\omega})$) in the denominator of Eq. (5.15) are approximated by the lineshape functions ($i\hbar\,\Sigma_{0\lambda+1\lambda}(\bar{\omega})$) as on the left-hand side of the equation, we can obtain an infinite number of coupled equations for the lineshape functions, which is similar to those of Shin et al. [22] based on the Nakajima profection operator method [23], of Lodder et al. [24] based on the proper connected diagram approach, and of Prasad [25] based on the coherent potential approximation approach obtained in the cyclotron resonance transition. If the quantities $\Xi_{i\perp}$ $(i=1,2)$ and the high order collision terms $i\hbar\ \Sigma_{1\lambda+1\lambda}(\bar{\omega})$ are neglected, Eq. (5.15) is reduced to Eq. (5.1a) obtained up to the second order terms for the weak coupling case. Real and imaginary parts of Eqs. (5.1), (5.15), and (5.16) give the lineshift and linewidth, respectively. By using Eqs. (4.46) and (4.47), the frequency-dependent conductivities for both intraband transition and intervalley and/or interband transitions are, respectively, given by

$$\begin{aligned}\mathrm{Re}\{\sigma_{yy}(\bar{\omega})\} &= (\hbar^2/\Omega)\sum_{\lambda}|j_{y\lambda+1\lambda}|^2\frac{f(E_{\lambda+1})-f(E_\lambda)}{E_{\lambda+1}-E_\lambda}\\ &\times\frac{\Gamma_{0\lambda+1\lambda}(\omega)}{[\hbar\omega-E_{\lambda+1}+E_\lambda-\hbar\ \tilde{\nabla}_{0\lambda+1\lambda}(\omega)]^2+\hbar^2\ \Gamma_{0\lambda+1\lambda}^2(\omega)]}\end{aligned}, \qquad (5.21)$$

$$\begin{aligned}\mathrm{Re}\{\sigma_{yy}(\bar{\omega})\} &= (\hbar^2/\Omega)\sum_{\lambda}|j_{y\lambda s'\lambda s}|^2\frac{f(E_\lambda^{s'})-f(E_\lambda^{s})}{E_\lambda^{s'}-E_\lambda^{s}}\\ &\times\frac{\Gamma_{0\lambda+1\lambda}(\omega)}{[\hbar\omega-E_\lambda^{s'}+E_\lambda^{s}-\hbar\ \tilde{\nabla}_{0\lambda s'\lambda s}(\omega)]^2+\hbar^2\ \Gamma_{0\lambda s'\lambda s}^2(\omega)]}\end{aligned}, \qquad (5.22)$$

where Re means "the real part of". Γ_0 and $\tilde{\nabla}_0$, respectively, can be calculated from Eq. (5.1) for a weak coupling case and from Eqs. (5.15) and (5.16) for a strong coupling case. The results of linewidth for intraband transition and intervalley and/or interband transitions in the case of weak coupling are, resepectively, given by

$$\begin{aligned}[\ \Gamma_{0\lambda+1\lambda}(\omega)]_{ph} &\equiv Im\{i\hbar\ \Sigma_{0\lambda+1\lambda}(\bar{\omega})\}\\ &= \pi\sum_q(1+n_q)[\sum_{\alpha(\neq\lambda+1)}(\gamma_q)_{\lambda+1\alpha}\{(\gamma_q^{+})_{\alpha\lambda-1}-(\gamma_q^{+})_{\alpha-1\lambda}j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}\\ &\quad\times\delta(\hbar\bar{\omega}-E_\alpha+E_\lambda-\hbar\omega_q)\\ &\quad+\sum_{\alpha(\neq\lambda)}\{(\gamma_q)_{\lambda\alpha}-(\gamma_q)_{\lambda+1\alpha+1}j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q^{+})_{\alpha\lambda}\\ &\quad\times\delta(\hbar\bar{\omega}-E_{\lambda+1}+E_\alpha+\hbar\omega_q)]\end{aligned}$$

$$
\begin{aligned}
&+\pi\sum_q n_q\Big[\sum_{\alpha(\neq\lambda+1)}(\gamma_q^+)_{\lambda+1\alpha}\{(\gamma_q)_{\alpha\lambda+1}-(\gamma_q)_{\alpha-1\lambda}j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}\\
&\times\delta(\hbar\bar{\omega}-E_\alpha+E_\lambda+\hbar\omega_q)\\
&+\sum_{\alpha(\neq\lambda)}\{(\gamma_q^+)_{\lambda\alpha}-(\gamma_q^+)_{\lambda+1\alpha+1}j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q)_{\alpha\lambda}\\
&\times\delta(\hbar\bar{\omega}-E_{\lambda+1}+E_\alpha-\hbar\omega_q)\Big]
\end{aligned}
\qquad (5.23)
$$

$$
\begin{aligned}
[\,\Gamma_{0\lambda s'\lambda s}(\bar{\omega})]_{ph} &\equiv Im\{i\hbar\,\Sigma_{0\lambda s'\lambda s}(\bar{\omega})\}\\
&=\pi\sum_q(1+n_q)\Big[\sum_{\alpha(\neq\lambda)}(\gamma_q)_{\lambda s'\alpha s'}\{(\gamma_q^+)_{\alpha s'\lambda s}-(\gamma_q^+)_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}\\
&\quad\times\delta(\hbar\bar{\omega}-E_\alpha^{s'}+E_\lambda^{s}-\hbar\omega_q)\\
&\quad+\{(\gamma_q)_{\lambda s\alpha s}-(\gamma_q)_{\lambda s'\alpha s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q^+)_{\alpha\lambda}\\
&\quad\times\delta(\hbar\bar{\omega}-E_\lambda^{s'}+E_\alpha^{s}+\hbar\omega_q)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_q\sum_{\alpha(\neq\lambda+1)}n_q\Big[(\gamma_q^+)_{\lambda s'\alpha s'}\{(\gamma_q)_{\alpha s'\lambda s'}-(\gamma_q)_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}\\
&\times\delta(\hbar\bar{\omega}-E_\alpha^{s'}+E_\lambda^{s}+\hbar\omega_q)\\
&+\{(\gamma_q^+)_{\lambda s\alpha s}-(\gamma_q^+)_{\lambda s'\alpha s'}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q)_{\alpha s\lambda s}\\
&\times\delta(\hbar\bar{\omega}-E_\lambda^{s'}+E_\alpha^{s}-\hbar\omega_q)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_q\sum_{\alpha(\neq\lambda)}(1+n_q)\frac{(E_\lambda^{s'}-E_\lambda^{s})-(E_\alpha^{s'}+E_\alpha^{s})}{E_\lambda^{s}-E_\alpha^{s}-\hbar\omega_q}(\gamma_q)_{\lambda s'\alpha s'}(\gamma_q^+)_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\\
&\times\{\delta(\hbar\omega-E_\alpha^{s'}+E_\alpha^{s})-\delta(\hbar\omega-E_\alpha^{s'}+E_\lambda^{s}-\hbar\omega_q)\}
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_q\sum_{\alpha(\neq\lambda)}(1+n_q)\frac{(E_\lambda^{s'}-E_\lambda^{s})-(E_\alpha^{s'}+E_\alpha^{s})}{E_\alpha^{s'}-E_\lambda^{s'}+\hbar\omega_q}(\gamma_q)_{\lambda s'\alpha s'}(\gamma_q^+)_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\\
&\times\{\delta(\hbar\omega-E_\alpha^{s'}+E_\alpha^{s})-\delta(\hbar\omega-E_\lambda^{s'}+E_\alpha^{s}+\hbar\omega_q)\}
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_q\sum_{\alpha(\neq\lambda)}n_q\frac{(E_\lambda^{s'}-E_\lambda^{s})-(E_\alpha^{s'}+E_\alpha^{s})}{E_\lambda^{s}-E_\alpha^{s}+\hbar\omega_q}(\gamma_q^+)_{\lambda s'\alpha s'}(\gamma_q)_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\\
&\times\{\delta(\hbar\omega-E_\alpha^{s'}+E_\alpha^{s})-\delta(\hbar\omega-E_\alpha^{s'}+E_\lambda^{s}+\hbar\omega_q)\}
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_q\sum_{\alpha(\neq\lambda)}n_q\frac{(E_\lambda^{s'}-E_\lambda^{s})-(E_\alpha^{s'}+E_\alpha^{s})}{E_\lambda^{s'}-E_\alpha^{s'}-\hbar\omega_q}(\gamma_q^+)_{\lambda s'\alpha s'}(\gamma_q)_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\\
&\times\{\delta(\hbar\omega-E_\alpha^{s'}+E_\alpha^{s})-\delta(\hbar\omega-E_\lambda^{s'}+E_\alpha^{s}-\hbar\omega_q)\}
\end{aligned}
\qquad (5.24)
$$

where the symbol Im in Eqs. (5.23) and (5.24) denotes the imaginary parts of the quantity. The lineshifts, $\hbar\,\widetilde{\nabla}_{0\lambda s'\lambda s}(\omega)[\equiv \mathrm{Re}\{i\hbar\,\Sigma_{0\lambda s'\lambda s}(\bar{\omega})\}]$ and and $\hbar\,\widetilde{\nabla}_{0\lambda+1\lambda}(\omega)[\equiv \mathrm{Re}\{i\hbar\,\Sigma_{0\lambda+1\lambda}(\bar{\omega})\}]$ can be calculated through a Kramers-Kronig relation :

$$\widetilde{\nabla}_{0}(\omega) = \frac{1}{\pi} P\int_{-\infty}^{\infty} \frac{\Gamma_{0}(\omega')}{\omega-\omega'}\, d\omega', \tag{5.25}$$

where $\Gamma_0(\omega')$ is given by Eqs. (5.23) and (5.24). To obtain Eqs. (5.23) and (5.24), we have used the Dirac identity

$$\lim_{s\to 0^+}(x\pm is) = P(1/x) \mp i\pi\delta(x), \tag{5.26}$$

where P denotes Cauchy's principle value integral.

By taking the real part and the imaginary part of Eqs. (5.15) and (5.16) for strong coupling case, we can obtain the linewidths ($\Gamma_{0\lambda s'\lambda s}(\omega)$, $\Gamma_{0\lambda+1\lambda}(\omega)$) and the lineshifts ($\widetilde{\nabla}_{0\lambda s'\lambda s}(\omega)$, $\widetilde{\nabla}_{0\lambda+1\lambda}(\omega)$) for both intraband transition and intervalley and/or interband transitions. The results of the linewidths for both intraband transition and intervalley and/or interband transitions are, respectively, given by

$$\begin{aligned}
\hbar\,\Gamma_{0\lambda+1\lambda}(\omega) =& \sum_q (1+n_q)\Big[\sum_{\alpha(\neq\lambda+1)} \\
&\times \frac{(\gamma_q)_{\lambda+1\alpha}\{(\gamma_q^{+})_{\alpha\lambda+1}-(\gamma_q^{+})_{\alpha-1\lambda}j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}\hbar\,\Gamma_{1\lambda+1\lambda}(\omega)}{[\hbar\omega-E_\alpha+E_\lambda-\hbar\omega_q+\Xi_{1+}+\hbar\,\widetilde{\nabla}_{1\lambda+1\lambda}(\omega)]^2+\hbar^2\,\widetilde{\Gamma}^2_{1\lambda+1\lambda}(\omega)} \\
&+\sum_{\alpha(\neq\lambda)} \frac{\{(\gamma_q)_{\lambda\alpha}-(\gamma_q)_{\lambda+1\alpha+1}j_{y\alpha+1\alpha}/j_{y\lambda+1\lambda}\}(\gamma_q^{+})_{\alpha\lambda}\hbar\,\Gamma_{1\lambda+1\lambda}(\omega)}{[\hbar\bar{\omega}-E_{\lambda+1}+E_\alpha+\hbar\omega_q+\hbar\,\widetilde{\nabla}_{1\lambda+1\lambda}(\omega)]^2+\hbar^2\,\widetilde{\Gamma}^2_{1\lambda+1\lambda}(\omega)}\Big] \\
&+\sum_q n_q\Big[\sum_{\alpha(\neq\lambda+1)} \\
&\times \frac{(\gamma_q^{+})_{\lambda+1\alpha}\{(\gamma_q)_{\alpha\lambda+1}-(\gamma_q)_{\alpha-1\lambda}j_{y\alpha\alpha-1}/j_{y\lambda+1\lambda}\}\hbar\,\Gamma_{1\lambda+1\lambda}(\omega)}{[\hbar\omega-E_\alpha+E_\lambda+\hbar\omega_q+\Xi_{1-}+\hbar\,\widetilde{\nabla}_{1\lambda+1\lambda}(\omega)]^2+\hbar^2\,\widetilde{\Gamma}^2_{1\lambda+1\lambda}(\omega)}
\end{aligned} \tag{5.27}$$

$$\begin{aligned}\hbar\,\Gamma_{0\lambda s'\lambda s}(\omega) &= \sum_{q}\sum_{\alpha(\neq\lambda)}(1+n_q) \\ &\times\Big[\frac{(\gamma_q)_{\lambda s'\alpha s}\{(\gamma_q^{+})_{\alpha s'\lambda s'}-(\gamma_q^{+})_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}\hbar\,\Gamma_{1\lambda s'\lambda s}(\omega)}{[\hbar\omega-E_\alpha^{s'}+E_\lambda^{s}-\hbar\omega_q+\Xi_{3+}+\hbar\,\widetilde{\nabla}_{1\lambda s'\lambda s}(\omega)]^2+\hbar^2\,\widetilde{\Gamma}^2_{1\lambda s'\lambda s}(\omega)} \\ &+\frac{\{(\gamma_q)_{\lambda s\alpha s}-(\gamma_q)_{\alpha s'\lambda s'}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q^{+})_{\alpha s\lambda s}\hbar\,\Gamma_{1\lambda s'\lambda s}(\omega)}{[\hbar\omega-E_\lambda^{s'}+E_\alpha^{s}+\hbar\omega_q+\Xi_{4-}+\hbar\,\widetilde{\nabla}_{1\lambda s'\lambda s}(\omega)]^2+\hbar^2\,\widetilde{\Gamma}^2_{1\lambda s'\lambda s}(\omega)}\Big] \\ &+\sum_{q}\sum_{\alpha(\neq\lambda)}n_q\Big[\frac{(\gamma_q^{+})_{\lambda s'\alpha s}\{(\gamma_q)_{\alpha s'\lambda s'}-(\gamma_q)_{\alpha s\lambda s}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}\hbar\,\Gamma_{1\lambda s'\lambda s}(\omega)}{[\hbar\omega-E_\alpha^{s'}+E_\lambda^{s}+\hbar\omega_q+\Xi_{3-}+\hbar\,\widetilde{\nabla}_{1\lambda s'\lambda s}(\omega)]^2+\hbar^2\,\widetilde{\Gamma}^2_{1\lambda s'\lambda s}(\omega)} \\ &+\frac{\{(\gamma_q^{+})_{\lambda s\alpha s}-(\gamma_q^{+})_{\alpha s'\lambda s'}j_{y\alpha s'\alpha s}/j_{y\lambda s'\lambda s}\}(\gamma_q)_{\alpha s\lambda s}\hbar\,\Gamma_{1\lambda s'\lambda s}(\omega)}{[\hbar\omega-E_\lambda^{s'}+E_\alpha^{s}-\hbar\omega_q+\Xi_{4+}+\hbar\,\widetilde{\nabla}_{1\lambda s'\lambda s}(\omega)]^2+\hbar^2\,\widetilde{\Gamma}^2_{1\lambda s'\lambda s}(\omega)}\Big],\end{aligned} \qquad (5.28)$$

where $\widetilde{\nabla}_1$ and Γ_1 in Eqs. (5.27) and (5.28), respectively, are the real part and imaginary part of the high order self-energy ($-i\hbar\,\Sigma_{1\lambda s'\lambda s}(\omega)$) in Eq. (4.45). If Γ_1 and $\widetilde{\nabla}_1$ of Eqs. (5.27) and (5.28) for a strong coupling are approxiamted for \$ Γ_0 and $\widetilde{\nabla}_0$, respectively, we obtain an infinite number of coupled equations for the linewidths and the frequency shifts, which is similar to those of Suzuki [26]. It is interesting to note that the δ-functions in Eqs. (5.23) and (5.24) express the law of energy conservation in one-phonon collision (emission and absorption) processes. The energy-conserving δ functions imply that when electron undergoes a collision by absorbing the energy from the incident photon, its energy can only change by an amount equal to the energy of a phonon involved in the transition. This in fact leads to the optically dected magnetophonon resonance. In addition, we see from Eqs. (2.14)-(2.17), (5.23), (5.24), (5.27), and (5.28) that the linewidths which is closely related to the ODMPR are given as a function of temperature, the tilt angle and the strength of magnetic field, the incident photon frequency, the difference in the effective mass between initial and final states of the intervalley or interband scattering by phonon, and the involved phonon energy.

V. Conclusions

So far we have presented a theory of optically detected magnetophonon resonance arising from various transitions including intraband transition and intervalley transition as well as interband transition due to the interaction with phonons in semiconductors in tilted magnetic fields . The perturbation has been dealt with two techniques based on the Mori-type method of calculation. One is a closed-form representation which is applicable to the weak scattering case and the other is a continued-fraction form representation which is applicable to the strong scattering case. It is interesting to note that the continued-fraction representation can be expressed by both the infinite expansion of the finite continued-fraction order and the infinite continued-fraction representations.

For sufficiently weak electron-phonon coupling, our results of lineshape function for the intraband transition are identical with Choi et al' result [17] obtained by Argyres-Sigel's projection operator method [18] and with Ryu et al.' result [19] obtained by Kawabata's projection operator method [20] in the cyclotron transition. However, the present results for interband transition have extra terms together with those of Choi et al. [21] and Yi et al. [14] obtained in the magnetooptical transition problem, as shown in Eq. (5.1b). It seems that the main difference results in the approximation that L_T of Eq. (4.17) is replaced by L_t under the assumption that $P_0'(L_e+L_{ph})X=0$ for any operator X. For strong electron-phonon coupling, the results obtained for the intraband transition are similar to those of some other authors [22-26] otained by using the renormalization of the superpropagators to include many body coherence effects in the cyclotron resonance transition problem. These results are given in the iterative manner while our result is given in the continued-fraction representation. Moreover, the results obtained by the interband transition are similar to those of Suzuki et al. [26] obtained by resolvent superoperator method. Unfortunately, we don't know any results to be compared with our results for intervalley transition. Thus we may claim that applying the Mori-type projection approach we can obtain the lineshpe functions and linewidths closely related to the ODMPR arising from various transitions including the intraband transition and the intervalley transition, as well as the interband transition.

From the δ-functions in Eqs. (5.23) and (5.24) giving the law of energy conservation in one-phonon collision (emission and absorption) processes, strong oscillations of the linewidth due to various transitions including the intraband transition in a 3D system such as GaAs and the intervalley transition in materials having many-valley structure such as Si and Ge, as well as the interband transition in zero-gap materials such as HgTe are expected, which indicate that the ODMPR should also be observed experimentally in such bulk semiconductors. The detailed works for the understanding of the physical characteristics on the ODMPR due to such transitions will be left for publication in a separate paper.

In conclusion, we have obtained a general form of frequency-dependent magnetoconductivity, by using the Mori-type projection operator technique presented by one of the present authors, and presented the explicit expressions of the lineshape function and the linewidth related to the optically detected magnetophonon resonances due to various transitions including intraband, intervalley, and interband transition in bulk semiconductors, which are expressed in two different ways for a weak coupling and an arbitrary and/or strong one.

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<Abstract>

Theoretical study on the optical detection of magnetophonon resonance in semiconductors in tilted magnetic fields

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On the basis of the Kubo formalism for linear response, a theory of optically detected magnetophonon resonance in tilted magnetic fields is presented for the Faraday configuration. The frequency-dependent magnetoconductivity of the system is evaluated by using the Mori-type projection technique. The general lineshape functions which are applicable to both a weak coupling and an arbitrary and/or strong electron-phonon coupling cases are introduced in two different ways. Explicit expressions of the lineshape functions and linewidths closely related to the optically detected magnetophonon resonance are obtained for various transitions including intraband, intervalley, and interband transitions. The results obtained here are in good agreement with those available in the literature.

Keywords : Frequency-dependent magnetoconductivity, Magnetophonon resonance, Optically detected magnetophonon resonance